METROSCOPE 4.0 GENRALIZED DESIGN AND EQUATION SPECIFICATIONS AS IMPLEMENTED AT PORTLAND METRO

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Introduction

The following report comes in two basic sections. The first section deals with the description and documentation of MetroScope in a verbal and schematic format. We describe the model's operation in terms of decision-making modules. We define the decision process of the consumer agents and we elaborate on the arguments that go into each of the decisions. This first section provides a general understanding of the logic and rationale of the MetroScope modeling procedure. The intent of this section is that it be accessible to the motivated reader without the necessity of deciphering substantial tracts of mathematical notation.

The second section provides the equation details. This second section includes a verbal summary of the equation structure – equation by equation without delving into specific mathematical notation. We also provide an explicit mathematical representation of the model simulation system as it is presently used. Those versed in mathematical notation may find the second approach more accessible and precise.

SECTION ONE

Urban Simulation Models – Explanation and Some History

MetroScope belongs to a class of models that are generically called "urban simulation models". What these models purport to do is to estimate current and future values of one or more measurable urban attributes. Certainly, the most common and well known of these urban simulation models is the 4 step transportation model. This model developed in the 1950's from even earlier hand and calculator based traffic engineering approaches measures, estimates and forecasts the attributes of urban traffic. Originally, the transportation model sought to estimate total auto trips over a large geography. This information converted into "desire lines" could then be interpreted by experienced traffic engineers and used to determine the timing and size of the future need for roads and bridges. Over a 50 year period, the transportation model has seen numerous improvements so that now the model estimates and forecasts detailed trip data for a variety of trip modes, purposes, times, and destinations at a block level geography over highly detailed transportation networks.

Growing use of the transportation model during the late 50's and early 60's underscored the need for similar methods for estimating the attributes of land use; particularly population and employment. Growing use of computers, data bases and the application of neoclassical economics to problems of urban location quickly lead to proposals to combine transportation and land use models into "Integrated Transportation and Land Use" models wherein many measurable attributes of urban areas were to be estimated and forecast in one model. These proposals saw the development of 3 models in the 1960's that purported to be "Integrated Transportation and Land Use" models. These models were one: the Herbert – Stevens Model, two: EMPIRIC and three: the Lowry Spatial Interaction Model.

Of the three models only the Herbert – Stevens Model[1] formally embodied micro-economic theory and explicit market clearing. Unfortunately, it relied on very incomplete and simplified housing demand and supply equations and reflected a very poor knowledge of urban real estate. Limited computer capacity and over reliance on the linear programming techniques of the early 60's hampered development of the model and finally lead to its abandonment.

EMPIRIC[2] represented an attempt to simulate urban development in a purely econometric format with numerous statistical equations estimated from cross –sectional data for urban areas. The impetus for EMPIRIC was the success of the national accounting models that simulated the macro measures of the economy such as GNP, national income and employment growth. EMPIRIC quickly came to grief when the estimated equations combined into a system failed to produce outputs on urban accounts that made any sense. Failure to have an explicit urban accounting structure similar to the National Income and Product Accounts (NIPA) provided a classic early example of what came to be known famously as the GIGO (garbage in, garbage out) principle. EMPIRIC too, was abandoned without implementation.

The Lowry Spatial Interaction Model[3] was an adaptation of the "law of social gravity" introduced into the literature by J.Q. Stewart in a 1947 *Geographical Review* paper[4] and later elaborated by G.K.Zipf[5] and several other authors. The Lowry model emphasized the link between distance or travel time to work and residential location. It also recognized the link between certain types of employment and household location. Given an external estimate of "basic" employment both households and non basic employment could be located as some function of distance. Neither supply, nor prices, nor markets, entered into the model. The strength of the Lowry model was that it fit cross sectional data well; could be easily estimated and required relatively little data to calibrate and operate. The weakness of the model was that it only responded to changes in travel time; consequently leaving out most of the information relevant to urban development. However, of the 3 model types; the Lowry model and its descendants were implemented and continue to be used in some MPO's to the present day.

Where does MetroScope fit into the above integrated model typology? In terms of model structure, MetroScope is most like the Herbert-Steven's model. It does harbor within its set of equations a fairly straightforward neoclassical demand and supply structure with a requirement that we find a price for each location and real estate type that matches demand and supply. However, in terms of how the equations are developed and their role in the model, MetroScope borrows heavily from the other model types as well. For instance, the Herbert-Steven's model did not use statistical equations for preference and choice; nor did it account for such factors as neighborhood quality; nor the tradeoff between travel time to work and housing price. Using statistical equations more in the spirit of the Lowry Model and to a much less extent EMPIRIC,

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[1] J. D. Herbert, B.H. Stevens, *A Model for the Distribution of Residential Activity in Urban Areas,* **Journal of Regional Science, 2.** (February 1960), pp. 21 – 36.
[2] D.M. Hill, D. Brand, W. B. Hansen, *Prototype Development of a Statistical Land Use Prediction Model for the Greater Boston Region,* **Highway Research Record, No 114,** (1965), pp. 51 – 70.
[3] I.S. Lowry, **A Model for Metropolis,** The Rand Corporation RM-4033-RC, (1964).
[4] J.Q. Stewart, *Empirical Mathematical Rules Concerning the Distribution and Equilibrium of Population,* **Geographical Review, XXXVII,** July 1947, pp. 461 – 485.
[5] G.K. Zipf, *The Hypothesis of the Minimum Equation at Unifying Social Principle,* **American Sociologicial Review, XXII,** December 1947, pp 627 –650.

MetroScope accounts for taste and preference factors in real estate choice plus "averages" the responses in recognition of factors and circumstances that cannot be accounted for.

The historical review above gives important background to a key MetroScope feature: it iterates in search of an equilibrium, market-clearing solution. For many models fitting of statistical equations to observed data constitutes almost all the work of model building. Once that chore is completed the analyst runs the model once through; all the outputs from all model equations are assumed to be consistent with one another. Common econometric models of the national and regional economies work that way as do urban simulation models in the Lowry or EMPIRIC tradition. Recent urban micro-simulation models, even those with stochastic (random) elements, typically also eschew market equilibrium. Models in the Herbert-Steven's tradition [6] such as MetroScope seek equilibrium.

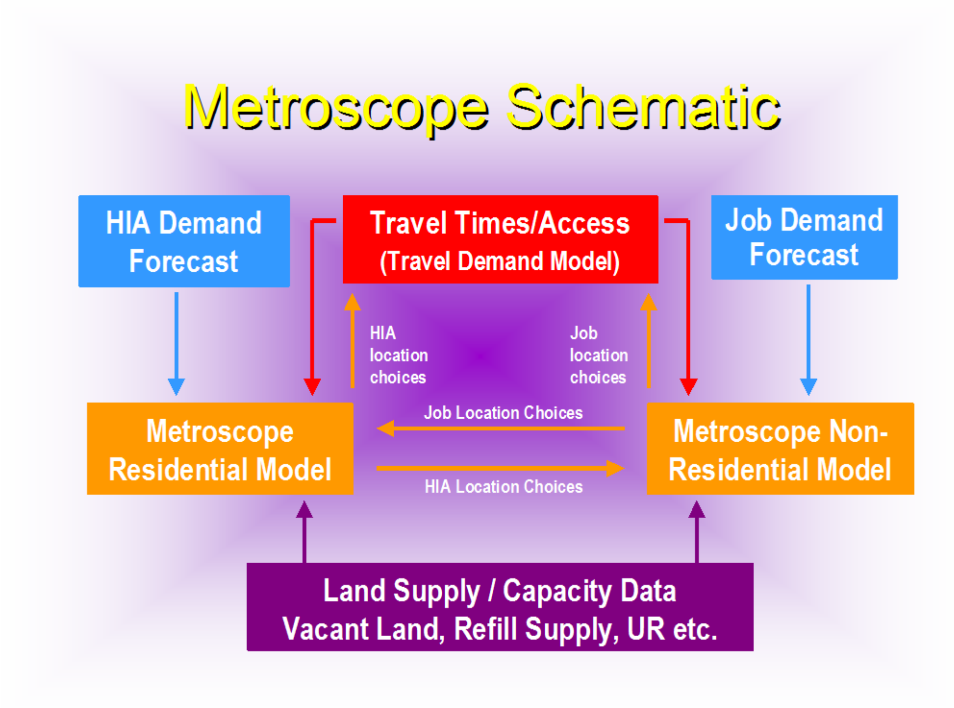
In MetroScope (as in most such models) all the demand and supply equations in addition to whatever other variables are included also include the price variable by location and real estate type. Demand responds negatively to an increase in price and supply responds positively to an increase in price. Consequently, supply and demand do not automatically match one another. The model must adjust prices iteratively for supply and demand to match for all locations and real estate types and the market to clear. In MetroScope statistical fitting of equations constitutes about 20 – 25% of the work. Establishing the equation structure, calibration to base year initial conditions and insuring the model iterates to a stable, consistent equilibrium in each forecast period constitute most of the MetroScope development effort.

Schematic Approaches to a Verbal Understanding of MetroScope

In the verbal description section we are following the convention of providing an overall view of MetroScope by using a simple diagram that portrays the basic computational modules as boxes and uses arrows to denote information flow between the modules. Figure A, next, shows the most general schematic.

[6] MEPLAN, TRANUS, PECAS, MUSSA, Metrosim, and MetroScope come to mind as models that require solution (some type of iterative procedure) for prices that achieve market clearing in each time period. Papers on these models may be obtained at: 2nd Oregon Symposium on Integrating Land Use and Transport Models, Portland, OR, July 2000 (CD ROM: Oregon DOT, William J. Upton: william.j.upton@odot.state.or.us). We should also point out that the travel demand model is also an iterative, equilibrium model in that origin-destination travel times are required to be consistent across the generation, mode choice, distribution and network assignment modules.

FIGURE A: METROSCOPE GENERAL SCHEMATIC



Blue Boxes (exogenous regional demand forecast)

Following the schematic shown above - the information contained in the blue boxes, "HIA demand forecast" (persons per household (H), household income bracket (I), head householder age (A)) and "job demand forecast" (jobs are categorized by available NAICS categories), are provided as regional control totals every 5 years and are forecast by Metro's regional econometric model. On the HIA (demographic) side we presently have regional estimates for every 5 year forecast period for 5 classes of household size, 5 classes of age of household head, 8 classes of household income and 2 classes of school age children present (yes or no). The combinatorics of these the HIA categories adds up to 400 classes or consumer segments which the model need process through the residential demand module in each 5 year period.[7] On the employment demand side the numbers are much more modest as we presently use 15 employment classes (i.e., agriculture-mining, retail trade, medical-social service, etc.). Important here is that these variables are supplied at the regional level by the econometric model and MetroScope then finds a location within the region for each of them.

Purple Box (exogenous regional supply)

The other source of external information that MetroScope requires is located in the purple box at the bottom of the diagram. The data that enter the model here at each 5 year forecast interval are of two varieties. The first variety amounts to the basic initial land use conditions of the region being modeled. These data include the amount of vacant land in each location by zoning class that is or will be available for building during the forecast period. The data also

[7] Actually there are 360 effective classes, since 1 person household with children present is logically precluded from occurring though the 2000 Census lists about 65 such households (legally emancipated children).

include the land already considered developed that will be available during the forecast period through infill and redevelopment. In addition, the model recognizes urban renewal (i.e., development subsidies or location specific infusion of reinvestment funding) efforts that produce additional resources and building that would not otherwise appear in a strictly market solution.

Beyond these initial period accounts, the MetroScope land and policy inputs are updated every 5 years to reflect Urban Growth Boundary (UGB) expansions and the creation (or deletion) of new (or existing) urban renewal districts. The model inputs also provide for zone changes, changes in development charges (SDC's) and changes in subsidy levels. Once land data are entered into the model, MetroScope incorporates a system of accounts for land consumption by location, zoning and land source (i.e., vacant, infill and redevelopment, urban renewal, UGB add and rural).

Red Box

The red box contains the travel demand model. The travel demand model consists of 4 different modules. These are (1) the trip generation module, (2) the mode split module, (3) the distribution module and (4) the network assignment module[8]. As presently implemented the travel demand modules use HIA data by TAZ (census tract in this case) that is collapsed into 64 classes and employment data by TAZ that is collapsed into 3 classes. Output data from the transportation demand model are travel times or logsums between all TAZ pairs. These data are then used in both the residential and nonresidential real estate modules as one of the variables used to determine demand for location in particular zones. Information is exchanged during each "major iteration" between the real estate modules and the travel demand modules of MetroScope within a forecast period. Since the travel demand model is well documented elsewhere, we do not describe this model in any more detail within this report.

Orange Box 1 – MetroScope Residential Real Estate Model

Since both the econometric model and travel demand model have been well documented elsewhere, the two orange boxes delimiting the residential and nonresidential real estate models comprise the focus of this MetroScope documentation. Skipping over computation details at this point the residential model has the following major components.

Demand Module -

- Assign households by HIAK class and place of employment to owner or renter.
    - Assign households by HIAK class, place of employment, and owner/renter to 8 housing consumption bins.
        - Assign households by HIAK class, place of employment, owner/rent, and housing consumption bin to single family or multi-family housing type.
            - Assign households by HIAK class, place of employment, owner/rent, housing consumption bin, and housing type to one of 494 location choices.

[8] The traffic assignment module uses proprietary software licensed from INRO and is not subject to the open source license. MPO's using MetroScope should be prepared to provide their own network assignment module and whatever coding would be required (usually very little) to convert distribution module output into a form useable for their particular assignment module.

- Compute for households by HIAK class, place of employment, owner/rent, housing consumption bin, housing type, and location how much they are paying for the housing choice.

A conceptual flow of the nonresidential model is described next.

Supply Module –

- Determine for each of 494 locations (R zones corresponding to 2010 census tracts) the amount of residential capacity available in a forecast period by zoning class.
- Determine for each of 494 locations, consumption bin and zone class the cost of producing a house.
- Determine for each location consumption bin and zone class whether the demand price is high enough to support production (i.e., exceeds production cost).
- For all feasible production determine supply output by location, consumption bin and zone class and compute land consumption by location and zone class.
- Subtract land used by location and zone class from available supply and determine total land supply available for the next period.
- Add new production to vintage (existing) housing stock by location and consumption bin for each location to determine total supply available.

Equilibrium Iteration –

- For each location zone determine total supply and total demand
- For zones with excess supply reduce the "location price" by a specified increment. For zones with excess demand increase the "location price" by a specified increment.
- Go to next iteration until maximum number of iterations is reached.[9]
- Pass the total households summed by employment zone (groups of R zones) to the nonresidential model.
- When the residential model and nonresidential models have exchanged data a set number of times, stop the run and pass the HIAK data by location (R Zone – TAZ) to the travel demand model.

Orange Box 2 – MetroScope Nonresidential Real Estate Model

The nonresidential model works in much the same manner as the residential real estate model. As presently configured the nonresidential model uses fewer zones than the residential model – 72 E zones as compared to 494 R zones. This does not reflect a limitation in computational ability of the software and hardware. Rather it reflects a realistic appreciation of the heterogeneity of nonresidential real estate and the resultant very high level of margin for error engendered from small zone sizes. The major modules of the nonresidential model we list below. For the nonresidential model we presently use 14 employment categories that we assign to seven real estate types. These real estate types are 1) manufacturing, 2) warehousing, 3) flex space, 4) retail trade/services, 5) general office, 6) medical/social and government, 7) residential areas. Compared to the residential model the nonresidential model is more aggregate and has

[9] We use a set number of iterations rather than reducing sum squared error below a specified level. This insures that we always get the same result given identical inputs at the start of a run.

fewer equations to explain the consumer non-residential demand and its components. Aggregation and detail level is driven from the diversity of nonresidential uses, the large level of data error and an accompanying limitation of the level of knowledge about nonresidential real estate transactions on both the demand and supply sides. This module runs very fast because of its simpler system design.

A conceptual flow of the nonresidential model is described next.

Nonresidential Demand

- Determine by NAICS, real estate type and location the demand for nonresidential real estate denominated in square feet.
- Determine by NAICS, real estate type and location the demand for employment denominated in BLS employees.

Nonresidential Supply

- Determine for 72 E zones the amount of nonresidential capacity available by zoning category (FAR class) by 3 land source classes of industrial, commercial and residential.
- Determine for 72 E zones, land source and FAR class the cost per square foot of producing the 7 real estate types.
- Compare cost of construction to demand price for each of the 7 real estate types by E zone and FAR class to determine amount of square footage to be built in each E zone by real estate type.
- For each E zone and real estate type add the additional square footage to the vintage (existing) square footage to determine total supply available.

Equilibrium Iteration

- For each E zone and real estate type compare total supply square footage to total demand square footage.
- For E zones and real estate types with excess supply decrease the location price a specified amount. For E zones and real estate types with excess demand increase the location price a specified amount.
- Go to next iteration, stop when the maximum number of iterations is reached.
- Pass the total employment by Ezone to the residential model.
- When the nonresidential and residential models have exchanged household and employment data a set number of times, stop and pass the employment data by type to the travel demand model.

Outside the Box - Internal Iterations and Outputs

A couple of things are not explicit in the schematic. The first thing is the number of iterations between modules in a given 5 year forecast simulation period. All the modules iterate internally for a set number of iterations or until certain stability conditions have been met. Each real estate module is set to iterate internally for a set number of times (presently 25 iterations) and then exchange data with other half of the MetroScope model. Data exchanges between the residential and non-residential modules occur twice after which the residential and nonresidential halves of the MetroScope model are considered consistent with one another. Data from both modules are sent to the travel demand model (TDM) where trips are iterated through the distribution and assignment modules for a set number of iterations. These travel times then provide the travel times for the next iterations of the real estate modules for the 5 year forecast period. The land use modules are then run through their iteration sequence for one more cycle. When complete the real estate model outputs are sent to the travel model for a final set of iterations for the travel model and the outputs for the forecast period are considered complete. Since all models within MetroScope work on a set number of iterations, MetroScope has the property of producing identical outputs given identical inputs in repeated tests. **A small change in inputs or iteration steps results in different output. Consequently, all policy and investment changes produce a measured effect.** While perhaps old-fashioned, deterministic models do have an advantage over stochastic models[10] when you are trying to trace the socio-economic impact of adjusting forecast inputs and policy input assumptions.

Beyond the arrows pointing in various directions between boxes in Figure A, we make no explicit reference to output. MetroScope produces three types of output. These are:

- A very large number (100's of millions) of ephemeral outputs; which are overwritten during each iteration of the real estate and travel models. Some of these data are of interest for particular purposes and may be written to permanent files on a custom basis.
- Final iteration data that are used by other MetroScope modules and used in reports for each 5 year forecast period. These data are written out to a "data store" in CSV format and available for conversion to Access, Excel or other data manipulation format.
- "Post processor" data that are created from the output files that provide special reports on redevelopment and infill, growth in centers, infrastructure cost, average commute distance, density, land use consumption by source, annual housing and transportation costs, etc.

In addition to tabular output we also have constructed a "MetroScopeViewer" that displays and compares one or two scenarios at a time.

[10] While perhaps more like reality, stochastic models do have the property that you never get the same result twice (dependent on model complexity and/or resolution level). This makes error detection or policy analysis tedious for small errors or testing of land use and investment policies implemented at a financially realistic level.

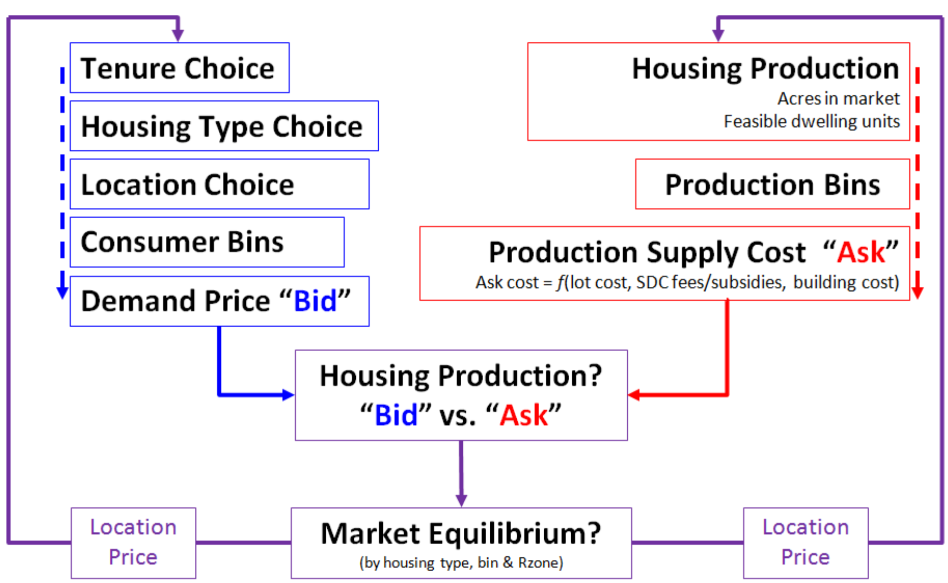
MetroScope Residential Model

The residential model uses a 494 Rzone (for residential zone) system of the year 2010 census tracts within MetroScope's 7-county coverage area. There are no limitations on the number of zones save computation time, volume of output and the increasing level of data and margin of error associated with tabulating smaller zones.

The MetroScope residential model recognizes 4 classes of real estate. Owner Single Family (OSF), Owner Multi-family (OSF), Renter Single Family (RSF) and Renter Multi-family (RMF) Single family vs. multi-family is based on tax lot ownership – if you own your roof and your land (house, townhouse) then that is considered single family. Otherwise, if the tax lot is shared ownership (apartment, condo), then that is considered multi-family.

In the residential demand model, we have a hierarchical system of choice. First a tenure choice is made; then a housing type choice, then a housing location choice. Parameter estimates are constrained such that the chance of a location change is always much higher than the chance of a housing type change, and likewise, the chance of a housing type change is greater than a tenure change.

This procedure yields an interpretable "bid rent" function that allows computation of willingness to pay. In the residential supply module, we calculate the cost to build a new dwelling unit in each zone class and value class (also called consumer / consumption bin). By adjusting prices, we iterate the model until market equilibrium is achieved.



Residential Demand – Tenure Choice

The first choice made by the household is the tenure. The number of households by tenure by HIAK for each Ezone is given by:

$$HH^{Own}_{HIAK,Ezone} = \left[HH^{Total}_{HIAK,Ezone}\right] x \left[P^{Own}_{HIAK,Ezone}\right]$$
$$HH^{Rent}_{HIAK,Ezone} = \left[HH^{Total}_{HIAK,Ezone}\right] x \left[P^{Rent}_{HIAK,Ezone}\right]$$

Where the total number of households (owners + renters) is:

$$HH^{Total}_{HIAK,Ezone} = \left[\left(Emp_{Ezone} \big/ TotalEmp\right) \times \left(TotalHH_{HIAK}\right)\right]$$

where TotalEmp and TotalHH are given by the regional forecast as regionwide growth totals for employment and households, respectively. The probability of the tenure choice is given by:

$$P^{Own}_{HIAK,Ezone} = U^{Own}_{HIAK,Ezone} / (1 + U^{Own}_{HIAK,Ezone}) \qquad P^{Rent}_{HIAK,Ezone} = 1 - P^{Rent}_{HIAK,Ezone}$$

The utility U of owner tenure is expressed by HIAK, and Ezone is derived from the binary choice logit equation (with a similar equation for renters):

$$U^{Own}_{HIAK,Ezone} = exp\begin{bmatrix} (Ln\, A_o + b_1 Ln(Age_{HIAK}) + b_2 Ln(Age^2_{HIAK}) + \\ b_3 Ln(Income_{HIAK}) + b_4 Ln(Income^2_{HIAK}) + \\ b_5 Ln(HHsize_{HIAK}) + b_6 (Kids_{HIAK}) + \\ b_7 Ln(Wt.LocPrice^{Rent}_{Ezone}) + \\ b_8 Ln(Wt.LocPrice^{Own}_{Ezone}) + \\ b_9 Ln(AccessIdx^{Tenure}_{Ezone}) \end{bmatrix}$$

Table 1: Parameter values for tenure choice equation - ownership

parameter	value
a0	-1.9
b1	-0.65
b2	0.49
b3	-1.79
b4	0.1447
b5	0.699849
b6	0.29966
b7	-0.75
b8	1.2
b9	1

The weighted location prices (Wt. Loc Price) are used in two terms of the utility equation, one to represent the choice (in this case, own), and the other to represent the substitute (rent).

Location prices by each of the 4 housing types are weighted by the Ezone-to-Rzone travel time:

$$Wt.\,LocPrice_{Ezone}^{HouseType} = \frac{\sum_{Rzone}[(LocPrice_{Rzone}^{HouseType} \times DU_{Rzone}^{HouseType})/(TravelTime_{Ezone,Rzone})]}{\sum_{Rzone}[DU_{Rzone}^{HouseType})/(TravelTime_{Ezone,Rzone,})]}$$

where LocPrice is the residential location price in the current model iteration. The location price by tenure is then the combined weights of each of two housing types: Owner price from OSF and OMF; Renter price from RSF and RMF (note the difference in superscripting of Wt.LocPrice). . .

$$Wt.LocPrice_{Ezone}^{Own} = \frac{\sum_{Rzone}[(Wt.LocPrice_{Rzone}^{OSF} \times DU_{Rzone}^{OSF})+(Wt.LocPrice_{Rzone}^{OMF} \times DU_{Rzone}^{OMF})]}{\sum_{Rzone}[(DU_{Rzone}^{OSF})+(DU_{Rzone}^{OMF})]}$$

$$Wt.LocPrice_{Ezone}^{Rent} = \frac{\sum_{Rzone}[(Wt.LocPrice_{Rzone}^{RSF} \times DU_{Rzone}^{RSF})+(Wt.LocPrice_{Rzone}^{RMF} \times DU_{Rzone}^{RMF})]}{\sum_{Rzone}[(DU_{Rzone}^{RSF})+(DU_{Rzone}^{RMF})]}$$

The access index (AccessIdx) takes into account the travel time between residents in each Rzone and their jobs in each Ezone:

$$AccessIdx_{Ezone}^{Own} = \sum_{Ezone}\sum_{Rzone}[DU_{Rzone}^{OSF})/TravelTime_{Ezone,Rzone}] + \sum_{Ezone}\sum_{Rzone}[DU_{Rzone}^{OMF}/TravelTime_{Ezone,Rzone}]$$

$$AccessIdx^{Rent}_{Ezone} = \sum_{Ezone}\sum_{Rzone}[DU^{RSF}_{Rzone})/TravelTime_{Ezone,Rzone}] + \sum_{Ezone}\sum_{Rzone}[DU^{RMF}_{Rzone}/TravelTime_{Ezone,Rzone}]$$

Residential Demand – Structure Type Choice

The next choice is by type. If you are an owner, you then choose between OSF and OMF; if you are a renter, then you choose between RSF and RMF.

The number of households by type and tenure HIAK for each Ezone is given by a similar set of equations as for the tenure choice:

$$HH^{Own,SFD}_{HIAK,Ezone} = \left(HH^{Own}_{HIAK,Ezone}\right)\times\left(P^{SFD,Own}_{HIAK,Ezone}\right) \qquad HH^{Rent,SFD}_{HIAK,Ezone} = \left(HH^{Rent}_{HIAK,Ezone}\right)\times\left(P^{SFD,Rent}_{HIAK,Ezone}\right)$$

$$HH^{Own,MFD}_{HIAK,Ezone} = \left(HH^{Own}_{HIAK,Ezone}\right)\times\left(P^{MFD,Own}_{HIAK,Ezone}\right) \qquad HH^{Rent,MFD}_{HIAK,Ezone} = \left(HH^{Rent}_{HIAK,Ezone}\right)\times\left(P^{MFD,Rent}_{HIAK,Ezone}\right)$$

with the probability of each housing type choice given by:

$$P^{SF,Own}_{HIAK,Ezone} = U^{SF,Own}_{HIAK,Ezone}/(U^{SF,Own}_{HIAK,Ezone} + U^{MF,Own}_{HIAK,Ezone}) \qquad P^{SF,Rent}_{HIAK,Ezone} = U^{SF,Rent}_{HIAK,Ezone}/(U^{SF,Rent}_{HIAK,Ezone} + U^{MF,Rent}_{HIAK,Ezone})$$

and the utility for each of the four type and tenure classes is given by this expression:

$$P^{MF,Own}_{HIAK,Ezone} = U^{MF,Own}_{HIAK,Ezone}/(U^{SF,Own}_{HIAK,Ezone} + U^{MF,Own}_{HIAK,Ezone}) \qquad P^{MF,Rent}_{HIAK,Ezone} = U^{MF,Rent}_{HIAK,Ezone}/(U^{MF,Rent}_{HIAK,Ezone} + U^{SF,Rent}_{HIAK,Ezone})$$

$$U^{HouseType,Tenure}_{HIAK,Ezone,Rzone} = \left[exp\left[\begin{array}{l} (Ln\, A_o + b_1 Ln(Age_{HIAK}) + b_2 Ln(Age^2_{HIAK}) + b_3 Ln(HHsize_{HIAK}) + b_4 Ln(Income_{HIAK}) + \\ b_5 Ln(Kids \times Income_{HIAK}) + b_8 Ln(PriceOfChoice_{Ezone}) + \\ b_9 Ln(PriceOfSubstitute_{Ezone}) + b_{10} Ln(TravelTimeWeight^{Type}_{Ezone}) \end{array}\right]\right]$$

Table 2: Parameter values for the structure type choice equations (4 sets)

parameter	osf	omf	rsf	rmf
a0	3.15	3.6	3.2	4.3
b1	0.075	-0.003	0.075	-0.003
b2	-0.003	0.065	-0.003	0.04
b3	0.7	-0.35	0.85	-0.25
b4	0.03	0.01	0.035	0.001
b5	0.025	-0.035	0.035	-0.02
b6	not used			
b7	not used			
b8	-0.75	0.75	-0.65	0.95
b9	0.75	-0.75	0.75	-0.5
b10	1	1	1	1

OSF = owner single family (e.g., 1-unit attached or 1-unit detached dwelling unit)
OMF = owner multi-family (e.g., condominium)
RSF = renter single family (e.g., for rent 1-unit attached or 1-unit detached dwelling unit)
RMF = renter multi-family (e.g., apartment)

The terms for the price of choice and the price of the substitute depend on the type and tenure

Owners: $Choice = Wt.\ Loc\ Price^{OSF}$; $Substitute = Wt.\ Loc\ Price^{MSF}$

Renters: $Choice = Wt.\ Loc\ Price^{RSF}$; $Substitute = Wt.\ Loc\ Price^{RSF}$

The individual location price equations are the same as what was used to calculate the weighted prices in the tenure choice equations:

$$Wt.Loc\,Price_{Ezone}^{Tenure,Type} = \frac{\sum_{Rzone}[(Loc\,Price_{Rzone}^{Tenure,Type} \times DU_{Rzone}^{Tenure,Type})/(TravelTime_{Ezone,Rzone})]}{\sum_{Rzone}[DU_{Rzone}^{Tenure,Type})/(TravelTime_{Ezone,Rzone,})]}$$

$$TravelTimeWeight_{Ezone}^{Tenure,Type} = \sum_{Ezone}\sum_{Rzone}[DU_{Rzone}^{Tenure,Type}/TravelTime_{Ezone,Rzone}]$$

Similarly, the travel time weight equations are the same as in the tenure choice equations:

$$TravelTimeWeight_{Ezone,year}^{HouseType} = \sum_{Ezone}\sum_{Rzone}[DU_{Rzone,year}^{HouseType})/TravelTime_{EzoneRzone,year}]$$

Residential Consumption Bins (or Value Classes)

At this point, we introduce the concept of residential "consumer / consumption bins". This is a mapping of each market segment (KHIA is same as HIAK, depends on sorting order of household characteristics) to one of eight housing value levels, such that the households in each segment only consider housing relevant to their own buying power. Put another way, we restrict households who can only afford to pay in lower price consumption bins to residential segments they can afford and at the same time segment high income households to residential consumer bins that match up with higher quality (priced) buyer expectations.

The first step is to assign each KHIA segment is to determine the average OSF housing price for each segment, then to use that price to determine a sort order for the KHIA segments. In current practice, this is done exogenously, using data from the Census/ACS.

$$KHIA\,Sort\,Order_{\;1-400} = \left(N_1, N_2, N_3,\ \ldots\ , N_{398}, N_{399}, N_{400}\right) \quad \text{such that:}$$

$$Lowest\,SFD\,Price = Avg\,House\,Price\,(N_1);\ Highest\,SFD\,Price = Avg\,House\,Price\,(N_{400})$$

The second step is to assign each of the sorted KHIA segments to one of the eight consumption bins, by tenure, so that each bin has roughly the same number of households. This is done for each iteration of the residential demand model.

$$\sum_{KHIAs\ in\ Bin} HH^{Own} = \tfrac{1}{8} \times \sum_{All\ KHIAs} HH^{Own} \qquad \sum_{KHIAs\ in\ Bin} HH^{Rent} = \tfrac{1}{8} \times \sum_{All\ KHIAs} HH^{Rent}$$

The bins also come into play when we compare the residential bid prices and ask costs, as described in the next section.

Residential Demand – Location Choice

The final household choice is to decide where to locate. The number of households by rzone and tenure/type is the sum of all households over the Ezones and HIAK (note: HIAK is same as KHIA nomenclature):

$$HH^{HouseType}_{Rzone} = \sum_{Ezone} \sum_{HIAK} HH^{HouseType}_{Rzone}$$

Where:

$$HH^{HouseType}_{HIAK,Ezone,Rzone} = HH^{HouseType}_{HIAK,Ezone} \times \left[\frac{\left[U^{HouseType}_{HIAK,Ezone,Rzone}\right] \times \left[FreqChoice^{HouseType}_{ConBin,Rzone}\right]}{\left[\sum_{Rzone}^{n} \left[U^{HouseType}_{HIAK,Ezone,Rzone}\right] \times \left[FreqChoice^{HouseType}_{ConBin,Rzone}\right]\right]} \right]$$

The frequency choice, FreqChoice, is the Rzone share for each consumer bin by housing type:

$$FreqChoice^{HouseType}_{ConBin,Rzone} = \left[DU^{HouseType}_{ConBin,Rzone}\right] / \left[\frac{1}{n_{Rzone}} \sum_{Rzone}^{n} DU^{HouseType}_{ConBin,Rzone}\right]$$

And the location choice utility U, another discrete choice logit equation, for each rzone, ezone type/tenure and HIAK is given by

$$U^{HouseType}_{HIAK,Ezone,Rzone} = exp\left[\begin{array}{l} (Ln\, A_o + b_1 Ln(TravelTime_{Ezone,Rzone}) + b_2 Ln(TravelTime^2_{Ezone,Rzone}) + \\ b_3 Ln(LocPrice_{Rzone}) + b_4 Ln(Kids \times TravelTime_{Ezone,Rzone}) + \\ b_5 Ln(Age \times TravelTime_{Ezone,Rzone}) + b_6 Ln(HHsize \times TravelTime_{Ezone,Rzone}) + \\ b_7 Ln(Income \times TravelTime_{Ezone,Rzone}) + \\ b_8 Ln(Income \times NeighborhoodScore_{Rzone}) + b_9 Ln(Kids \times KidsIdx_{Rzone}) \end{array}\right]$$

Where:

NeighborhoodScore is an index calculated from a linear regression on historical tax lot housing prices, controlling for such attributes as house size, lot size. Please see hedonic neighborhood model estimation study.

-- KidsIdx is a factor to increase the probability a household containing children where those households already exist.

Table 3: Parameter values for the location choice equations (4 sets)

parameter	osf	omf	rsf	rmf
a0	0	0	0	0
b1	-0.05	-0.1	-0.05	-0.06
b2	-0.125	-0.25	-0.135	-0.14
b3	-4.5	-3.9	-3.5	-4
b4	0.005	0.005	0.001	0.001
b5	-0.01	-0.015	-0.0125	-0.0125
b6	-0.02	-0.04	-0.02	-0.04
b7	0.005	0.001	0.001	0.001
b8	0.5	0.5	0.1	0.05
b9	1	1	1	1

OSF = owner single family (e.g., 1-unit attached or 1-unit detached dwelling unit)
OMF = owner multi-family (e.g., condominium)
RSF = renter single family (e.g., for rent 1-unit attached or 1-unit detached dwelling unit)
RMF = renter multi-family (e.g., apartment)

Residential Demand – “Bid” Price

The residential bid price for each housing unit is calculated for each tenure/type, KHIA segment, and Rzone:

$$HousePrice_{HIAK,Rzone}^{HouseType} =$$

$$\exp\left[\begin{array}{l} Ln(A_o) + b_1 Ln(LotSize_{HIAK}) + b_2 Ln(HouseSize_{HIAK}) + b_3 Ln(AccessIdx_{Rzone}) \\ + b_4 Ln(AttachedDUfactor^{HouseType}) + b_5 (NeighborhoodScore_{Rzone}) \\ + b_6 Ln(RealHousingPrice_{Rzone}^{HouseType}) \end{array}\right]$$

Where:

AccessIdx is an endogenous measure of a zone’s proximity to all other zones
AttachedDUfactor is a correction to add weight to multi-family housing types\
NeighborhoodScore is an endogenous index relating to relative desirability of a particular zone

Table 4: Parameter values for the residential bid price equations (4 sets)

parameter	osf	omf	rsf	rmf
a0	6.65	6.55	6.55	6.55
b1	0.149	0.149	0.149	0.149
b2	0.55	0.55	0.55	0.55
b3	0.05	0.055	0.055	0.075
b4	0	0.032	0	0.032
b5	1	1	1	1
b6	1	1	1	1

OSF = owner single family (e.g., 1-unit attached or 1-unit detached dwelling unit)
OMF = owner multi-family (e.g., condominium)
RSF = renter single family (e.g., for rent 1-unit attached or 1-unit detached dwelling unit)
RMF = renter multi-family (e.g., apartment)

$$LotSize_{HIAK,Rzone}^{HouseType} =$$

$$exp\left[\begin{array}{l} Ln(A_o) + b_1 Ln(HHsize) + b_2 Ln(Age) + b_3 Ln(Age)^2 + b_4 Ln(Income) \\ + b_5 (Kids) + b_6 Ln(Income \times Kids) \end{array}\right]$$

Table 5: Parameter values for lot size equation (4 sets)

paramete	osf	omf	rsf	rmf
a0	5.25	4.5	4.8	3.8
b1	0.2	0.2	0.2	0.2
b2	0.165	0.165	0.165	0.165
b3	-0.022	-0.022	-0.022	-0.022
b4	0.7	0.5	0.75	0.4
b5	0.15	0.15	0.15	0.15
b6	7.5E-06	0.000005	7.5E-06	0.000001

OSF = owner single family (e.g., 1-unit attached or 1-unit detached dwelling unit)
OMF = owner multi-family (e.g., condominium)
RSF = renter single family (e.g., for rent 1-unit attached or 1-unit detached dwelling unit)
RMF = renter multi-family (e.g., apartment)

$$HouseSize^{HouseType}_{HIAK,Rzone} =$$

$$exp\left[\begin{array}{l} Ln(A_0)+b_1 Ln(HHsize)+b_2 Ln(Age)+b_3 Ln(Age)^2+b_4 Ln(Income) \\ +b_5(Kids)+b_6 Ln(Income \times Kids) \end{array}\right]$$

Table 6: Parameter values for house size equation (4 sets)

paramete	osf	omf	rsf	rmf
a0	4.6	4.4	4.4	4
b1	0.2	0.2	0.35	0.2
b2	0.165	0.165	0.165	0.165
b3	-0.022	-0.022	-0.022	-0.022
b4	0.62	0.62	0.65	0.6
b5	0.15	0.1	0.15	0.05
b6	7.5E-06	0.000005	7.5E-06	0.000001

OSF = owner single family (e.g., 1-unit attached or 1-unit detached dwelling unit)
OMF = owner multi-family (e.g., condominium)
RSF = renter single family (e.g., for rent 1-unit attached or 1-unit detached dwelling unit)
RMF = renter multi-family (e.g., apartment)

$$RealHousingPrice^{HouseType}_{Rzone} = \frac{Loc.Price_{(CurrentYear)}}{Loc.Price_{(BaseYear)}}$$

Residential Supply – Land Accounting

The residential land supply acres available to the model in a given time period for each rzone is given by zoning class and housing type (single family and mult-family). Each zoning class is based on the underlying jurisdictional zoning, and defines a maximum and minimum lot size, and a base cost of construction per unit.

The residential model calculates supply and demand by type and tenure (note that the term "supply" by itself refers to dwelling units, while "land supply" refers to acres of buildable land).. However, the jurisdictional zoning typically gives no information about tenure, so the we make an exogenous assumption for each zoning class about how the gets split. For example, we expect units in areas zoned single-family to be mostly owner-occupied.
It is not expected that all of the eligible buildable land supply in a given model year will likely enter the marketplace, so we apply a "throttle", an endogenous multiplier to the total supply, adjusted as prices go up or down

$$AcresInMarket_{Rzone,ZoneClass}^{HouseType} = AcresAvailable_{Rzone,ZoneClass}^{HouseType} \times FractionInMarket_{Rzone}^{HouseType}$$

Where:

$$\left(FractionInMarket\right)_{Rzone}^{HouseType} = \left(BaseLandFraction\right)_{Rzone}^{HouseType} \times \left(Land\ PriceRatio\right)^{\beta}$$

$$LandPriceRatio_{Rzone}^{HouseType} = exp\,[\,A_o + b_1 ln(\,RealHousingPrice_{Rzone}^{Housetype}\,)]$$

$$RealHousing\,Price_{Rzone}^{HouseType} = \frac{Loc.Price\ (CurrentYear)}{Loc.Price\ (BaseYear)}$$

Table 7: Parameter values for the acres in market equation

parameter	value
β	1.5
A0	1.599
b1	1

Next we use a conversion of acres to dwelling units to calculate the dwelling unit supply capacity for each rzone, type, and zone class

$$DUCap_{Rzone,ZoneClass}^{HouseType} = AcresInMarket_{Rzone,ZoneClass}^{HouseType} \Big/ AvgLotSize_{Rzone,ZoneClass}^{HouseType}$$

where:

$$AvgLotSize^{HouseType}_{Rzone,ZoneClass} = [MedianSize_{ZoneClass} \times (LandQuantityRatio)]$$

such that:

$$Min.Size_{ZoneClass} \geq AvgLotSize^{HouseType}_{Rzone,Zone} \geq Max.Size_{ZoneClass}$$

Minimum, median, and maximum lot sizes are exogenous assumptions by zoning class.

Residential – "Ask" (Construction) Cost

The "ask" cost of building a new dwelling unit is a combination of the cost for the lot, the cost for the building, and any fees or subsidies per dwelling unit:

$$TotalProductionCost^{HouseType}_{Rzone,ZoneClass,Bin} =$$
$$LotCost^{HouseType}_{Rzone,ZoneClass,Bin} + BuildingCost^{HouseType}_{Rzone,ZoneClass,Bin} + Building\ Fee(or\ Subsidy)^{HouseType}_{Rzone}$$

The lot cost is comprised of an exogenous base lot cost by rzone and zone class, and an adjustment that is a function of capital costs and real housing price

$$LotCost^{HouseType}_{Rzone,ZoneClass} = BaseLotCost_{Rzone,ZoneClass} \times \left[1 + (PctChangeLotPrice)\right]$$

The total building cost equation is comprised of an exogenous construction cost per square foot, an average house size by consumption bin (calculated by the residential demand model, and an average lot size by consumption bin (also from the residential demand model). The last term is factor to take into account that houses on larger lots will be slightly less costly to build than those on smaller lots:

$$BuildingCost^{HouseType}_{Rzone,ZoneClass,Bin} = BaseCostSqft_{ZoneClass} \times AvgHouseSize^{HouseType}_{Rzone,Bin} \times \left[\frac{AvgLotSize^{HouseType}_{Rzone,Bin=1}}{AvgLotSize^{HouseType}_{Rzone,Bin}}\right]^{\beta}$$

Table 8: Parameter value of the building cost eqaution

parameter	value
β	0.1

The building fees, which are assumptions of system development charges (SDE) and subsidies, which are assumptions of a reduced cost per unit, such as in urban renewal district, are both exogenous values per unit, by rzone.

Residential – "Bid" vs. "Ask". . . to build or not to build . . .

Now we are able to compare the bid prices to the ask costs, and determine whether to build or to not build. Since the residential demand by bin does not have information about zone class, for each bin, we compare the bid price to the ask cost of every zone class:

$$\text{If:}\quad TotalProductionCost^{HouseType}_{Rzone,ZoneClass,Bin} \leq AvgHousePrice^{HouseType}_{Rzone,Bin}$$

Is Bid > Ask?

$$\text{Then:}\quad Total\ Feasible\ DU^{HouseType}_{Rzone,ZoneClass,Bin} = DUCap^{HouseType}_{Rzone,ZoneClass}$$

If yes, then build:

$$\text{Else:}\quad Total\ Feasible\ DU^{HouseType}_{Rzone,ZoneClass,Bin} = 0$$

If no, then don't build:

Here we note that in the above calculation, a feasible solution uses up all the capacity for the entire zone class. Feasible production in all eight consumption bins would then be eight times the actual available capacity. We correct for this by creating a utility for prorating the feasible supply and distributing it over the zone classes and consumer bins:

$$SupplyUtility^{HouseType}_{Rzone,ZoneClass,Bin} = \left(\frac{AvgHousePrice^{HouseType}_{Rzone,Bin}}{TotalProductionCost^{HouseType}_{Rzone,ZoneClass,Bin}} \right)^{\beta} \times DUCap^{HouseType}_{Rzone,ZoneClass,Bin}$$

where β is a factor which reflects the fact that dwelling units with a larger profit margin will be produced in excess of those with a smaller profit margin.

The actual new supply for each zone class is then

$$New\,DU^{HouseType}_{Rzone,ZoneClass,Bin} = \left[\frac{SupplyUtility_{ZoneClass,Bin}}{\sum_{bin} SupplyUtility_{ZoneClass,Bin}} \right] \ x\ DUCap^{HouseType}_{Rzone,ZoneClass,Bin}$$

The new supply is summed over the bins and zone classes, and the result is used to determine the market equilibrium. . .

$$NewDU^{Hou\sin gType}_{Rzone} = \sum_{Bin,ZoneClass} NewDU^{Hou\sin gType}_{Rzone,ZoneClass,Bin}$$

Residential Supply and Demand: Conditions for Market Equilibrium

Now that we have run both the supply and demand modules in the current iteration, we test for market equilibrium, and adjust location prices for the next model iteration until supply and demand match.

For each rzone and type/tenure, we seek to minimize the quantity:

$$Minimize: \quad Sum\,of\ Squares = \sum_{Rzone,HousingType} \left(Total\ Demand - Total\ Supply \right)^2$$

The total supply is the sum of the previous year's supply and the new supply built in the current iteration of the current year:

$$TotalSupply^{HousingType}_{Rzone,CurrentYear} = TotalSupply^{HousingType}_{Rzone,PreviousYear} + NewDU^{HousingType}_{Rzone,CurrentYear}$$

The total demand is calculated in the demand module, summed over the consumption bins:

$$Total\ Demand^{HousingType}_{Rzone} = \sum_{Bin} Demand^{HousingType}_{Rzone,Bin}$$

Finally, we adjust the location prices. If supply is greater than demand, then we reduce prices, so that it less cost-effective to build and easier to buy. Likewise, if supply is less than demand, then we increase prices, so that it is more cost-effective to build and more difficult to buy.

$$LocationPrice^{HousingType}_{Rzone,Iter.N+1} = LocationPrice^{HousingType}_{Rzone,Iter.N} + PriceChange^{HousingType}_{Rzone}$$

where:

$$PriceChange^{HousingType}_{Rzone} = A_0 \times \left(Total\ Demand^{HousingType}_{Rzone} - Total\ Supply^{HousingType}_{Rzone} \right)$$

In current practice, supply and demand do not converge exactly for all Rzones and housing types, but experience has shown that the model is close to equilibrium after a fixed number of iterations.

MetroScope Non-Residential Model

Draft 2016-09-23

The MetroScope non-residential real estate model can be characterized as a “limited equilibrium” land development simulation & forecasting model with market clearing demand and supply modules.

Initial demand estimates are computed as a “shift-share” with the final solution adjusted for price and access indices. The demand-side module forecasts demand by Ezones for 14 employment classes and 7 real estate types. The supply-side module forecasts development supply by Ezones for 3 land sources (i.e., commercial, industrial and residential zones) and 7 real estate types – (1)general industrial, (2) warehousing/distribution, (3) tech/flex space, (4) retail, (5) office and (6) institutional – government and medical spaces, (7) residential.

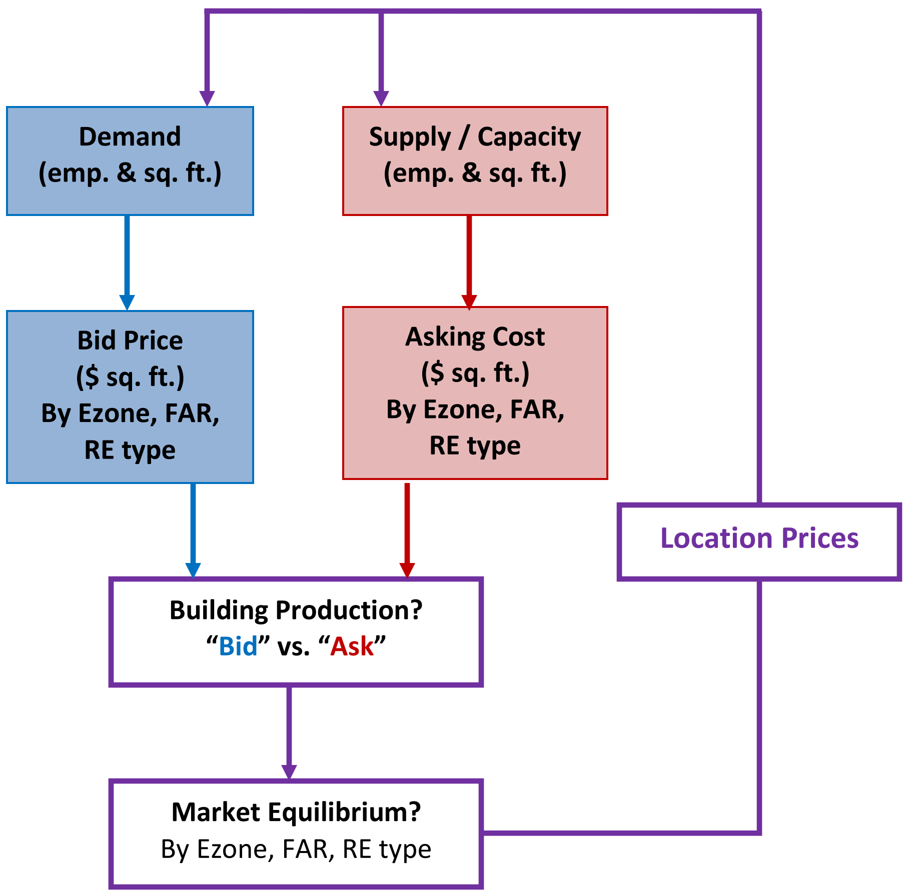
Equilibrium is sought in each Ezone for each of 7 real estate types by iterative price adjustments. Market clearing is achieved when Ezone level supply and demand (by real estate types) are equal at a given price.

The nonresidential model uses 72 Ezones which are aggregations of census tracts. Like the residential model, there is no restriction on the number of Ezones within the framework of the MetroScope model. The upper limit on number of zones is established by computational time, volume and complexity of output and the tolerance of the model for data error.

Compared to the residential model, employment data are far more heterogeneous and subject to various data errors in employer geocoding. Large zones minimize these errors. Also, since nonresidential zoning and locations are relatively limited in extent, the presence of nonresidential real estate is much more discontinuous as zone sizes decrease. These discontinuities pose calibration challenges to the model as otherwise identical zones are totally different in regard to employment location.

As in the residential model, the number of simultaneous equations that must be solved during each iteration of the non-residential model is determined by the number of zones multiplied by the number of real estate types. For the nonresidential model, there are72 Ezones times 7 real estate types, yielding 504 equations that are solved for location prices in each five year period.

Non-Residential Model – Schematic of the Market-Clearing Process



Emp. = employment
Sq. ft. = square feet
Ezone = MetroScope employment zones (census tract groupings)
FAR = floor area ratio
RE type = real estate type
Location price = index to ratchet supply and demand up or down depending upon market equilibrium conditions

Non-Residential Demand

First we estimate demand by Ezone, the 14 employment types and the 7 real estate types. The demand equations are structured to be similar to "shift and share" type equations but with variables measuring relative prices and accessibilities included in demand calculations.

The train of logic here is to start with a region-wide employment estimate by type; determine the share per the base year calibration that would be located in one of the 7 real estate types and then convert that employment figure to square footage demanded using the base year calibration for square feet per employee by the 14 employment types and the 7 real estate types. The access indices act to determine the base calibration year share in a particular Ezone while the various direct price elasticity's and cross-price elasticity's vary amounts by Ezone, employment type and real estate type.

Regional job growth by employment type is allocated to each of 7 real estate types by square feet per employee and then varies these quantities as a function of the location price for that real estate type in that Ezone and the competing location prices of all other real estate types in that Ezone.

$$
\begin{aligned}
&RawSqFt_{Ezone,EmpType}^{RealEstateType} = \\
&\quad \left[EmploymentTotal_{Region,EmpType}\right] \times \left[BaselineEmpDistribution_{EmpType}^{RealEsateType}\right] \times \\
&\quad \left[BaseSqFtperEmp_{EmpType}^{RealEstateType}\right] \times \left[Location\,Price_{Ezone,t}^{RealEstateType}\right]^{-\beta} \times \\
&\quad \prod_{i=1}^{6}\left[Location\,Price_{Ezone}^{RealEstateType}\right]^{\pm\alpha} \times \left[Location\,Price_{Ezone}^{RealEstateType}\right]^{-\mu} \times \\
&\quad \left[\begin{array}{l} AccessIDX_{Ezone,EmpType}^{HH} \times \delta_{EmpType}^{HH} + AccessIDX_{Ezone,EmpType}^{AllEmp} \times \delta_{EmptType}^{AllEmp} + \\ AccessIDX_{Ezone,EmpType}^{SameEmpType} \times \delta_{EmpType}^{SameEmpType} \end{array}\right]
\end{aligned}
$$

Where:

"Employment Total" are values passed from the regional economic forecast.

“Baseline Emp Distribution” and “Base SqFt per Emp” are exogenous inputs.

“Baseline Emp Distribution” allocates Employment Total to job type by real estate type.

“Baseline Sqft Per Emp” converts the jobs forecast into square footage demand by real estate type by job type and ezone.

“Location Price” are endogenous and vary the terms of sq. ft. per employee as a function of location price by real estate type over time.

β is the direct sq. ft. elasticity by real estate type (adjusts Base SqFt per Emp) Controls the change in square foot per employee from the base square foot per employee observed for the region in the calibration period.

α is the matrix of cross price elasticities by real estate types (adjusts Base SqFt per Employee) Here we make the standard assumption of symmetry with all the cross-elasticities summing to zero.

μ is the direct price elasticity by employment class (adjusts the baseline employment distribution for employment types) – derived from calibration.

δ weights the contribution of each of the three access indices [AccessIDX], described below. The three access indices sum to one.

The three access indices described in the above equation allocates the percentage share of each employment type that is to be allocated to the Ezone in tandem with the various real estate location prices. (see: “Appendix 2 – Exogenous Employment Demand Model Parameters” for job allocation, square foot density, and price elasticity parameters and other employment equation input terms.)

Access to Households:

$$AccessIDX^{HH}_{Ezone,EmpType} = RawAccessIDX^{HH}_{Ezone,EmpType} \, / \sum_{Ezone=i}^{72} RawAccessIDX^{HH}_{Ezone,EmpType}$$

Access to All Employment:

$$AccessIDX^{TotalEmp}_{Ezone,EmpType} = RawAccessIDX^{TotalEmp}_{Ezone,EmpType} \, / \sum_{Ezone=i}^{72} RawAccessIDX^{TotalEmp}_{Ezone,EmpType}$$

Access to Same Employment Type:

$$AccessIDX^{SameEmpType}_{Ezone,EmpType} = RawAccessIDX^{Same\backslash EmpType}_{Ezone,EmpType} \, / \sum_{Ezone=i}^{72} RawAccessIDX^{SameEmpType}_{Ezone,EmpType}$$

Using a base value of sq. ft. per employee, modified by price, we calculate new employment in each Ezone from the estimates square footage of real estate demanded by this equation:

$$RawEmp_{Ezone,EmpType}^{RealEstateType} = RawSqFt_{Ezone,EmpType}^{RealEstateType} \Big/ [\, BaseSqFtperEmp \times Location\,Price^{-\beta}\,]$$

Since we must match the regional employment controls by class, we calculate an adjustment factor based on the employment summed by employment class, and then apply that factor to the employment for each Ezone and real estate type:

$$AdjustFactorEmp_{Ezone,EmpType} = EmploymentTotal_{Region,Emptype} \Big/ \sum_{Ezone} \sum_{RealEstateType} RawEmp_{EmpType}$$

$$AdjustedEmp_{Ezone,EmpType}^{RealEstateType} = AdjustFactorEmp_{Ezone,EmpType} \times RawEmp_{Ezone,EmpType}^{RealEstateType}$$

Finally, we apply the same factor to the sq. ft. demand to get the final adjusted demand:

$$AdjustedSqFt_{Ezone,EmpType}^{RealEstateType} = AdjustFactorEmp_{Ezone,EmpType} \times RawSqFt_{Ezone,EmpType}^{RealEstateType}$$

Non-Residential Supply Model – "Bid vs. Ask"

"Land Class" has 3 sources: (Industrial, Commercial & Residential) from which 7 real estate types compete for land.

Industrial Land – Manufacturing, Warehousing, Tech/Flex Space

Commercial Land – Retail/Service, General Office, Medical/Government

Residential Land – Employment Uses on residential-zoned land

While real estate types are restricted to its land class, employment classes may occupy any type of real estate space subject to market clearing. Employment on residential land is a special case, however. The supply of residential acres is predetermined in advance, and does not compete with any other real estate types.

Instead of deriving a hedonic bid price in the non-residential demand module, as is done in the residential demand model, the market price "bid" and the market cost "ask" are simultaneously calculated within the non-residential supply module.

"Bid" prices are calculated at the real estate type level; "Ask" prices at the land class level.

"Bid" Price

Bid prices are calculated by real estate type, Ezone, and FAR

$$MarketPricePerSqFt_{Ezone,FARclass}^{REtype} = \left[BaseMarketPriceSqFt_{Ezone}^{REtype} + MarketParam^{REtype} \times ln\left(AvgFAR_{FARclass}\right)\right] \times RealPrice_{Ezone}^{REtype}$$

Subject to the constraint:

$$Market\,PricePerSqFt_{Ezone,FARclass}^{REtype} \geq Min\,PricePerSqft^{REtype}$$

And where:

$$RealPrice_{Ezone}^{REtype} = \frac{LocationPrice_{Ezone,t}^{REtype}}{BaseYearLocationPrice_{Ezone,t=0}^{REtype}}$$

Location price is endogenous and is an iterative calculation from the market clearing computations of the non-residential model. Base Market Price per SqFt, Average FAR, and Minimum Price per SqFt are exogenous assumptions. (see: "Appendix 3 – Exogenous Bid Price Parameters" for parameter values and input assumption terms)

"Ask" Cost

Construction Costs:

Construction costs (also known as asking cost) are denoted by the 3 land classes, Ezone and FAR, given by:

$$ConstructionCostperSqFt^{Landclass}_{Ezone,FARclass} =$$
$$\left(BaseCapitalCost^{Landclass}_{Ezone,FARclass} + BaseLandCost^{Landclass}_{Ezone,FARclass} \times Wt.RealPrice^{REtype}_{Ezone}\right) \times \left(\frac{MarketFAR}{AverageFAR}\right)$$

The construction cost is defined as capital costs plus land costs, modified by market effects. Parameter values and other input assumptions are in the appendix (see: Appendix 4 – Exogenous Land and Building cost parameters for the land and building cost equation inputs and parameters.) These three components are described as:

Capital Costs:

$$BaseCapitalCostPerSqft^{Landclass}_{Ezone,FARclass}$$
$$= BaseCapitalCostPerSqft^{Landclass}_{Ezone} + CapitalCostParam^{Landclass} \times ln\left(AvgFAR_{FARclass}\right)$$

Subject to the constraint:

$$BaseCapitalCostperSqFt^{Landclass}_{Ezone,FARclass} \geq MinCapitalCostPerSqft^{Landclass}$$

The base capital cost, minimum capital cost, capital cost parameter, and average FAR are exogenous model input assumptions. (see: Appendix – Exogenous Demand Model Parameters)

Land Costs:

$$BaseLandCostPerSqFt^{Landclass}_{Ezone,FARclass}$$
$$= BaseLandCostPerSqFt^{Landclass}_{Ezone} + LandCostParam^{Landclass} \times ln\left(AvgFAR_{FARclass}\right)$$

Subject to the constraint:

$$BaseLandCostPerSqft_{Ezone,FARclass}^{Landclass} \geq MinLandCostPerSqft_{Ezone}^{Landclass}$$

The base land cost, minimum land cost, land cost parameter, and average FAR are exogenous assumptions. (see Appendix)

Market-Adjusted FAR

The market adjusted FAR is calculated by:

$$MarketFAR_{Ezone,FARclass}^{Landclass} = AverageFAR_{FARclass} \times \left(LandPriceRatio_{Ezone}^{LandClass}\right)^{\delta}$$

Subject to the constraint:

$$MarketFAR_{Ezone,FARclass}^{LandClass} \geq MaxFAR_{FARclass}$$

Where:

Average FAR is an exogenous default value for each FAR class range

--- δ is a land/capital substitution parameter, by land class

The land price ratio in the market FAR equation above is given by

$$Land\ PriceRatio_{Ezone}^{LandClass} = exp\left[A_o + A_1 \times ln\left(Wt.Real\ Price_{Ezone}^{LandClass}\right) - A_2\right]$$

Where the real price for the particular land class is weighted by the supply of each of the real estate types that participate in it:

$$Wt.RealPrice_{Ezone}^{LandClass} = \frac{\sum_{RE=i,j,k}\left(RealPrice_{Ezone}^{RealEstateType} \times TotalSupply_{Ezone,PreviousYear}^{RealEstateType}\right)}{\sum_{RE=i,j,k} TotalSupply_{Ezone,PreviousYear}^{RealEstateType}}$$

Non-Residential Supply and Demand: Conditions for Market Equilibrium

Finally, for each Ezone, real estate type, and FAR class, we compare the bid price to the ask cost, and then decide whether to build or not build. The bid price is the market price per square foot by real estate type, as calculated in non-residential demand module, described earlier. The ask cost is the construction cost per square foot by land class, as calculated in the non-residential supply module described above in this section.

$if\ :MarketPriceSqFt^{RealEstateType}_{Ezone,FARclass} \geq ConstructionCostSqFt^{LandClass}_{Ezone,FARclass}$	**Is Bid > Ask?**
$then:\quad NewSqftSupply^{RealEstateType}_{Ezone,FARclass} = MarketSqftCapacity^{RealEstateType}_{Ezone,FARclass}$	**If yes, then build:**
$else:\quad NewSqftSupply^{RealEstateType}_{Ezone,FARclass} = 0$	**If no, then build no more**

Market Square Foot Capacity

The market capacity is the amount of non-residential square feet which is eligible for construction if it is cost effective to do so, using the above bid vs. ask decision.

The available land supply enters the model as buildable acres, by land class (commercial, industrial, residential), and FAR class. In practice, there does not exist complete spatial information of FARs on the buildable acres, so default FAR class assumptions are imposed over the buildable as an exogenous input assumption.

To reflect the idea that not all of the available acres will be offered to the market in a give model year, we apply a “throttle”, an exogenous base multiplier to the total supply. The current version of the non-residential model uses a default value of 0.4, but could just as well be a dynamic, priced-based multiplier, like that in the residential model. The throttle simply metes out the supply of non-residential acres for year t.

$$AvailableAcres^{Landclass}_{Ezone,FARckass} = TotalAcres^{Landclass}_{Ezone,FARclass,t} \times FractionInMarket^{Landclass}_{Ezone,t}$$

To convert the acres-based land supply into square footage-based real estate supply, we apply the market-adjusted FAR for each FAR class in the acres supply, as shown in the next equation. In practice, detailed FAR data of buildable land is not readily available, so default distributions are assumed and validation difficult. Industrial land will in general have lower FARs than commercial land, however.

$$MarketSqftCapacity_{Ezone,FARckass}^{Landclass} = MarketFAR_{Ezone,FARclass}^{Landclass} \times AvailableAcres_{Ezone,FARclass}^{Landclass} \;\; x \;\; 43{,}560$$

The final step is to apportion the land classes into their corresponding real estate types, weighted by the sqft demand by real estate type from the previous model year.

$$MarketSqftCapacity_{Ezone,FARclass}^{Re\ alEstateType=i} =$$

$$MarketSqftCapacity_{Ezone,FARclass}^{Landclass} \times \left[\frac{AdjustSqFt_{Ezone,FARclass}^{Re\ alEstateType=i}}{\sum_{REtype=i,j,k} AdjustSqFt_{Ezone,FARclass}^{Re\ alEstateType}} \right]$$

Where real estate types *i*,*j*,*k* participate in the given land use class.

Market Equilibrium

Now that both the supply and demand modules in the current non-residential are running, we test for market equilibrium, and adjust location prices for the next model iteration until supply matches demand.

One thing to note is the fact that the regional control totals are for employment by employment class, while the location prices are based on square feet by real estate type, so the precise dynamics of supply, demand, and employment are not entirely straightforward.

For each Ezone and real estate type, we seek to minimize the quantity:

$$Minimize: \quad Sum\,of\ Squares = \sum_{\substack{Ezone, \\ RealEstateType}} \left(Total\,Demand - Total\,Supply \right)^2$$

The total supply is the sum of the previous year's supply plus the new supply built in the current iteration of the current year:

$$TotalSupply_{Ezone,CurrentYear}^{RealEstateType} = TotalSupply_{Ezone,PreviousYear}^{RealEstateType} + NewSqftSupply_{Ezone,CurrentYear}^{RealEstateType}$$

The total demand calculated in the non-residential demand module, summed over the employment types:

$$Total\,Demand_{Ezone,CurrentYear}^{RealEstateType} = \sum_{EmpClass} AdjustedDemandSqft_{Ezone,EmpClass,CurrentYear}^{RealEstateType}$$

Finally, we adjust the location prices. If supply is greater than demand, then we reduce prices, so that it less cost-effective to build and cheaper to buy. Likewise, if supply is less than demand, then we increase prices, so that it is more cost-effective to build and more expensive to buy.

$$PriceChange_{Ezone}^{RealEstateType} = A_0 \times \left(Total\,Demand_{Ezone}^{RealEstateType} / Total\,Supply_{Ezone}^{RealEstateType} \right)$$

$$LocationPrice_{Ezone,Iter.N+1}^{RealEstateType} = LocationPrice_{Ezone,Iter.N}^{RealEstateType} + PriceChange_{Ezone,CurrentYear}^{RealEstateType}$$

In current practice, supply and demand do not converge exactly for all Ezones and real estate types, so the model is run for a fixed number of iterations.

Appendix 1 – Employment Categories by NAICS

(North American Industrial Classification System)

Empclass	Sector	NAICS
1	Agriculture, Mining and Forestry	NAICS 11, 21
2	Construction	NAICS 23
3	Education (private)	NAICS 61 (private)
4	Health and Social Services	NAICS 62
5	Manufacturing - Durable	NAICS 321, 331-333, 335-339
6	Manufacturing - High Tech.	NAICS 334
7	Manufacturing - Nondurable	NAICS 311-316,322-327
8	Other Services	NAICS 81
9	Professional and Business Services	NAICS 22, 51-56
10	Retail and Consumer Services	NAICS 44,45,71,72
11	Transportation, Warehousing	NAICS 48,49
12	Wholesale Trade	NAICS 42
13	Government - Education	NAICS 61 (public)
14	Government - non-Education	All other public

Appendix 2 – Exogenous Employment Demand Model Parameters (ref. p. 24)

Baseline allocation of employment to real estate type:

empclass	man	war	flex	ret	gen	med	res	sums to:
1	0.288238	0.139995	0.194569	0	0.377198	0	0	1
2	0.03	0.2	0.11	0.09	0.29	0	0.28	1
3	0	0.01	0	0.03	0.42	0.15	0.39	1
4	0	0.008777	0.017555	0.087774	0.193103	0.605016	0.087774	1
5	0.83	0	0.14	0.01	0.01	0	0.01	1
6	0.74	0	0.15	0	0.11	0	0	1
7	0.72	0.15	0.09	0.02	0.01	0	0.01	1
8	0	0.04	0.12	0.41	0.1	0.05	0.28	1
9	0.009894	0.019787	0.118723	0.148404	0.623723	0	0.079469	1
10	0	0.01	0.02	0.92	0.03	0	0.02	1
11	0	0.63	0.21	0.05	0.08	0	0.03	1
12	0.03	0.51	0.17	0.05	0.2	0	0.04	1
13	0	0	0	0	0.08	0.28	0.64	1
14	0	0.14	0.01	0.05	0.66	0.12	0.02	1

Baseline square feet per employee:

empclass	man	war	flex	ret	gen	med	res
1	600	1,250	500	450	450	450	100
2	100	400	200	100	100	100	100
3	1,200	1,200	1,000	1,200	350	1,200	100
4	400	600	400	600	350	450	100
5	650	750	700	450	350	350	100
6	400	600	700	450	350	350	100
7	700	800	700	450	350	350	100
8	400	600	400	450	350	350	100
9	400	600	400	450	350	350	100
10	400	600	350	350	350	350	100
11	800	3,300	1,000	500	450	450	100
12	400	1,400	500	500	450	450	100
13	1,200	1,200	1,000	1,200	350	1,200	100
14	600	800	600	450	350	600	100

α (cross price elasticity) parameter values:

empclass	retype	man	war	flex	ret	gen	med	res
1	man	-0.5	0.225	0.225	0.01	0.01	0.01	0.01
1	war	0.225	-0.5	0.225	0.01	0.01	0.01	0.01
1	flex	0.225	0.225	-0.5	0.01	0.01	0.01	0.01
1	ret	0.01	0.01	0.01	-0.07	0.01	0.01	0.01
1	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
1	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
1	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
2	man	-0.5	0.225	0.225	0.01	0.01	0.01	0.01
2	war	0.225	-0.5	0.225	0.01	0.01	0.01	0.01
2	flex	0.225	0.225	-0.5	0.01	0.01	0.01	0.01
2	ret	0.01	0.01	0.01	-0.07	0.01	0.01	0.01
2	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
2	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
2	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.5
3	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
3	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
3	flex	0.01	0.01	-0.715	0.225	0.225	0.225	0.01
3	ret	0.01	0.01	0.225	-0.715	0.225	0.225	0.01
3	gen	0.01	0.01	0.225	0.225	-0.715	0.225	0.01
3	med	0.01	0.01	0.225	0.225	0.225	-0.715	0.01
3	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
4	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
4	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
4	flex	0.01	0.01	-0.5	0.225	0.225	0.01	0.01
4	ret	0.01	0.01	0.225	-0.5	0.225	0.01	0.01
4	gen	0.01	0.01	0.225	0.225	-0.5	0.01	0.01
4	med	0.01	0.01	0.01	0.01	0.01	-0.35	0.15
4	resland	0.01	0.01	0.01	0.01	0.01	0.15	-0.35
5	man	-0.35	0.15	0.15	0.01	0.01	0.01	0.01
5	war	0.15	-0.35	0.15	0.01	0.01	0.01	0.01
5	flex	0.15	0.15	-0.35	0.01	0.01	0.01	0.01
5	ret	0.01	0.01	0.01	-0.07	0.01	0.01	0.01
5	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
5	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
5	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
6	man	-0.35	0.15	0.15	0.01	0.01	0.01	0.01
6	war	0.15	-0.35	0.15	0.01	0.01	0.01	0.01
6	flex	0.15	0.15	-0.35	0.01	0.01	0.01	0.01
6	ret	0.01	0.01	0.01	-0.07	0.01	0.01	0.01
6	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
6	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
6	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
7	man	-0.5	0.225	0.225	0.01	0.01	0.01	0.01
7	war	0.225	-0.5	0.225	0.01	0.01	0.01	0.01
7	flex	0.225	0.225	-0.5	0.01	0.01	0.01	0.01
7	ret	0.01	0.01	0.01	-0.07	0.01	0.01	0.01
7	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
7	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
7	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07

8	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
8	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
8	flex	0.01	0.01	-0.5	0.225	0.225	0.01	0.01
8	ret	0.01	0.01	0.225	-0.715	0.225	0.225	0.01
8	gen	0.01	0.01	0.225	0.225	-0.715	0.225	0.01
8	med	0.01	0.01	0.01	0.225	0.225	-0.5	0.01
8	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.25
9	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
9	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
9	flex	0.01	0.01	-0.5	0.225	0.225	0.01	0.01
9	ret	0.01	0.01	0.225	-0.715	0.225	0.225	0.01
9	gen	0.01	0.01	0.225	0.225	-0.715	0.225	0.01
9	med	0.01	0.01	0.01	0.225	0.225	-0.5	0.01
9	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.25
10	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
10	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
10	flex	0.01	0.01	-0.5	0.225	0.225	0.01	0.01
10	ret	0.01	0.01	0.225	-0.5	0.225	0.01	0.01
10	gen	0.01	0.01	0.225	0.225	-0.5	0.01	0.01
10	med	0.01	0.01	0.01	0.01	0.01	-0.07	0.01
10	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
11	man	-0.65	0.3	0.3	0.01	0.01	0.01	0.01
11	war	0.3	-0.65	0.3	0.01	0.01	0.01	0.01
11	flex	0.3	0.3	-0.65	0.01	0.01	0.01	0.01
11	ret	0.01	0.01	0.01	-0.25	0.1	0.1	0.01
11	gen	0.01	0.01	0.01	0.1	-0.25	0.1	0.01
11	med	0.01	0.01	0.01	0.1	0.1	-0.25	0.01
11	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
12	man	-0.35	0.15	0.15	0.01	0.01	0.01	0.01
12	war	0.15	-0.35	0.15	0.01	0.01	0.01	0.01
12	flex	0.15	0.15	-0.35	0.01	0.01	0.01	0.01
12	ret	0.01	0.01	0.01	-0.25	0.1	0.1	0.01
12	gen	0.01	0.01	0.01	0.1	-0.25	0.1	0.01
12	med	0.01	0.01	0.01	0.1	0.1	-0.25	0.01
12	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
13	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
13	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
13	flex	0.01	0.01	-0.07	0.01	0.01	0.01	0.01
13	ret	0.01	0.01	0.01	-0.16	0.01	0.1	0.01
13	gen	0.01	0.01	0.01	0.01	-0.07	0.01	0.01
13	med	0.01	0.01	0.01	0.1	0.01	-0.16	0.01
13	resland	0.01	0.01	0.01	0.01	0.01	0.01	-0.07
14	man	-0.07	0.01	0.01	0.01	0.01	0.01	0.01
14	war	0.01	-0.07	0.01	0.01	0.01	0.01	0.01
14	flex	0.01	0.01	-0.07	0.01	0.01	0.01	0.01
14	ret	0.01	0.01	0.01	-0.16	0.01	0.1	0.01
14	gen	0.01	0.01	0.01	0.01	-0.11	0.01	0.05
14	med	0.01	0.01	0.01	0.1	0.01	-0.2	0.05
14	resland	0.01	0.01	0.01	0.01	0.05	0.05	-0.19

β (direct square-foot elasticity) parameter values:

empclass	man	war	flex	ret	gen	med	res
1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
2	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
3	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
4	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1	-0.1
5	-0.05	-0.05	-0.05	-0.2	-0.2	-0.2	-0.2
6	-0.05	-0.05	-0.05	-0.2	-0.2	-0.2	-0.2
7	-0.05	-0.05	-0.05	-0.2	-0.2	-0.2	-0.2
8	-0.1	-0.1	-0.15	-0.2	-0.2	-0.1	-0.1
9	-0.1	-0.1	-0.15	-0.25	-0.25	-0.25	-0.1
10	-0.1	-0.1	-0.15	-0.25	-0.25	-0.25	-0.1
11	-0.05	-0.05	-0.05	-0.2	-0.2	-0.2	-0.2
12	-0.05	-0.05	-0.05	-0.2	-0.2	-0.2	-0.2
13	-0.05	-0.05	-0.05	-0.15	-0.15	-0.15	-0.05
14	-0.05	-0.05	-0.05	-0.15	-0.15	-0.15	-0.05

μ (direct price elasticity) parameter values:

empclass	value
1	-0.333
2	-0.5
3	-0.5
4	-0.5
5	-0.48967
6	-0.58617
7	-0.34478
8	-0.666
9	-0.666
10	-0.666
11	-0.666
12	-0.3787
13	-0.70102
14	-0.333

δ (access index weights) parameter values:

empclass	all emp	hh	empclass	sums to:
1	0.225039	0.031551	0.74341	1
2	0	0.038773	0.961227	1
3	0.034903	0	0.965097	1
4	0	0	1	1
5	0	0	1	1
6	0	0	1	1
7	0	0	1	1
8	0.512234	0	0.487766	1
9	0	0	1	1
10	0	0	1	1
11	0	0	1	1
12	0	0	1	1
13	0.0604	0.318079	0.621522	1
14	0	0	1	1

Appendix 3 - Exogenous Bid Price Parameters (ref. p. 27)

Market parameter by real estate type:

	man	war	flex	ret	gen	med	resland
Market Param.	-27.36	-21.96	-34.69	-21.63	40.507	40.507	-33.51
Min. Price per Sqft	15	15	15	15	15	15	15

FAR class parameters:

FAR class	Min FAR	Avg FAR	Max FAR
1	0	0.2	0.25
2	0.25	0.375	0.5
3	0.5	0.625	0.75
4	0.75	0.875	1
5	1	1.25	1.5
6	1.5	1.75	2
7	2	3.5	5
8	5	7.5	15

(note: the market price per square foot cost equation uses only "AvgFAR" variable in the table above; the min and max terms are shown only for illustrative purposes.)

Base Market Price per square foot by real estate type:

ezone	man	war	flex	ret	gen	med	res
1	74.9	52.82	97.18	214.49	239.61	239.61	191.46
2	69.67691	49.13664	90.40324	180.5893	201.739	216.1105	176.1961
3	67.25277	47.42712	87.258	159.0648	177.6937	200.2651	166.8647
4	52.9623	37.34938	68.71664	140.2737	156.7018	185.7148	152.3987
5	55.20615	38.93176	71.62796	166.5259	186.0286	205.8495	164.6534
6	60.63123	42.75756	78.66679	149.8789	167.4319	193.2435	157.7489
7	38.96004	27.47489	50.54923	151.9957	169.7967	194.8765	158.3845
8	50.08865	35.32286	64.98818	143.434	160.2323	188.214	156.914
9	54.63522	38.52914	70.8872	136.3554	152.3247	182.5846	156.1612
10	53.63866	37.82635	69.59419	134.8054	150.5932	181.3365	145.5306
11	45.3366	31.97169	58.82258	132.9822	148.5564	179.8609	145.005
12	44.05885	31.0706	57.16474	151.6673	169.4299	194.6238	140.1569
13	43.23188	30.48742	56.09178	131.4219	146.8133	178.5917	152.6404
14	49.06774	34.60291	63.66359	138.2865	154.4819	184.1317	149.0403
15	47.6221	33.58343	61.78792	137.1818	153.2478	183.2477	148.7987
16	40.6697	28.68055	52.76744	131.5679	146.9764	178.7107	144.1054
17	47.80637	33.71338	62.02701	136.2256	152.1797	182.4803	148.8755
18	53.282	37.57484	69.13144	133.8618	149.5391	180.5738	144.4072
19	42.97709	30.30774	55.7612	123.721	138.2106	172.2371	138.2689
20	50.08865	35.32286	64.98818	132.9066	148.472	179.7996	143.2102
21	41.03653	28.93925	53.2434	128.0553	143.0525	175.8326	141.2258
22	40.23563	28.37445	52.20426	124.3234	138.8835	172.7398	141.764
23	39.30943	27.72128	51.00254	127.951	142.936	175.7466	148.1741
24	37.91174	26.73562	49.18909	127.7642	142.7273	175.5926	148.5156
25	35.41802	24.97704	45.95358	79.36531	88.66018	131.959	102.0238
26	52.37817	36.93745	67.95875	157.9543	176.4531	199.425	174.6117
27	19.91702	14.04562	25.8416	57.03606	63.71584	108.2307	86.48157
28	43.4141	30.61593	56.3282	128.884	143.9783	176.5144	140.8087
29	44.53581	31.40696	57.78357	125.2068	139.8704	173.4752	148.6369
30	45.21638	31.88691	58.6666	119.1355	133.088	168.378	145.7816
31	43.68885	30.80968	56.68468	122.6941	137.0634	171.3779	127.3089
32	41.42782	29.21519	53.75108	115.1949	128.686	165.014	143.1381
33	42.38224	29.88825	54.9894	113.4094	126.6913	163.4745	136.7156
34	37.08734	26.15425	48.11947	98.10414	109.5936	149.8556	105.1127
35	38.91912	27.44603	50.49613	110.6856	123.6485	161.1074	133.3609
36	35.82611	25.26482	46.48307	107.2974	119.8635	158.1299	122.1585

37	35.41802	24.97704	45.95358	101.8223	113.7472	153.238	122.1585
38	41.828	29.49739	54.27029	121.4677	135.6934	170.3481	137.2822
39	48.39623	34.12935	62.79233	150.534	168.1638	193.7499	143.8822
40	32.31101	22.78595	41.92235	123.1018	137.5188	171.7194	132.4695
41	43.48073	30.66291	56.41465	115.1936	128.6845	165.0128	139.5119
42	37.61934	26.52942	48.80971	116.1993	129.808	165.8758	129.8187
43	44.84566	31.62547	58.1856	132.9956	148.5714	179.8718	144.0429
44	48.17091	33.97046	62.49998	130.85	146.1745	178.125	148.0866
45	46.62387	32.87948	60.49276	129.8465	145.0535	177.3042	147.8198
46	46.29432	32.64708	60.06519	134.0526	149.7521	180.7281	145.6667
47	53.34256	37.61754	69.21001	130.166	145.4104	177.5658	163.2997
48	48.35227	34.09835	62.73529	134.22	149.9392	180.8635	145.8208
49	42.08727	29.68023	54.60669	122.2766	136.597	171.0278	154.8762
50	47.27267	33.33702	61.33456	133.0987	148.6866	179.9555	160.9184
51	45.61484	32.1679	59.18357	123.4465	137.9039	172.0077	160.7614
52	45.40098	32.01709	58.90611	129.9496	145.1687	177.3886	159.1166
53	42.51181	29.97963	55.15752	126.0408	140.802	174.1676	155.4772
54	48.66294	34.31744	63.13837	124.4217	138.9934	172.8218	155.0928
55	39.86923	28.11606	51.72886	118.3526	132.2135	167.7133	140.6822
56	41.14171	29.01342	53.37986	119.3296	133.3049	168.5426	157.9648
57	40.60926	28.63793	52.68902	110.3473	123.2707	160.8118	140.2768
58	53.34256	37.61754	69.21001	130.166	145.4104	177.5658	163.2997
59	53.34256	37.61754	69.21001	130.166	145.4104	177.5658	163.2997
60	43.23188	30.48742	56.09178	131.4219	146.8133	178.5917	152.6404
61	41.14171	29.01342	53.37986	119.3296	133.3049	168.5426	157.9648
62	40.60926	28.63793	52.68902	110.3473	123.2707	160.8118	140.2768
63	46.62387	32.87948	60.49276	129.8465	145.0535	177.3042	147.8198
64	41.828	29.49739	54.27029	121.4677	135.6934	170.3481	137.2822
65	41.14171	29.01342	53.37986	120.6166	134.7426	169.6309	135.5433
66	48.17091	33.97046	62.49998	120.6166	134.7426	169.6309	135.5433
67	40.60926	28.63793	52.68902	120.6166	134.7426	169.6309	135.5433
68	37.61934	26.52942	48.80971	120.6166	134.7426	169.6309	135.5433
69	35.41802	24.97704	45.95358	120.6166	134.7426	169.6309	135.5433
70	37.06975	26.14184	48.09664	120.6166	134.7426	169.6309	135.5433
71	35.41802	24.97704	45.95358	120.6166	134.7426	169.6309	135.5433
72	35.41802	24.97704	45.95358	120.6166	134.7426	169.6309	135.5433

Minimum floor price per square foot by real estate type:

ezone	man	war	flex	ret	gen	med	res
1	4	4	4	4	4	4	4
2	4	4	4	4	4	4	4
3	4	4	4	4	4	4	4
4	4	4	4	4	4	4	4
5	4	4	4	4	4	4	4
6	4	4	4	4	4	4	4
7	4	4	4	4	4	4	4
8	4	4	4	4	4	4	4
9	4	4	4	4	4	4	4
10	4	4	4	4	4	4	4
11	4	4	4	4	4	4	4
12	4	4	4	4	4	4	4
13	4	4	4	4	4	4	4
14	4	4	4	4	4	4	4
15	4	4	4	4	4	4	4
16	4	4	4	4	4	4	4
17	4	4	4	4	4	4	4
18	4	4	4	4	4	4	4
19	2.5	2.5	2.5	2.5	2.5	2.5	2.5
20	2.5	2.5	2.5	2.5	2.5	2.5	2.5
21	2.5	2.5	2.5	2.5	2.5	2.5	2.5
22	2.5	2.5	2.5	2.5	2.5	2.5	2.5
23	2.5	2.5	2.5	2.5	2.5	2.5	2.5
24	2.5	2.5	2.5	2.5	2.5	2.5	2.5
25	2.5	2.5	2.5	2.5	2.5	2.5	2.5
26	4	4	4	4	4	4	4
27	2.5	2.5	2.5	2.5	2.5	2.5	2.5
28	4	4	4	4	4	4	4
29	4	4	4	4	4	4	4
30	4	4	4	4	4	4	4
31	4	4	4	4	4	4	4
32	4	4	4	4	4	4	4
33	4	4	4	4	4	4	4
34	2.5	2.5	2.5	2.5	2.5	2.5	2.5
35	2.5	2.5	2.5	2.5	2.5	2.5	2.5
36	2.5	2.5	2.5	2.5	2.5	2.5	2.5

37	2.5	2.5	2.5	2.5	2.5	2.5	2.5
38	4	4	4	4	4	4	4
39	4	4	4	4	4	4	4
40	4	4	4	4	4	4	4
41	2.5	2.5	2.5	2.5	2.5	2.5	2.5
42	2.5	2.5	2.5	2.5	2.5	2.5	2.5
43	4	4	4	4	4	4	4
44	4	4	4	4	4	4	4
45	4	4	4	4	4	4	4
46	4	4	4	4	4	4	4
47	4	4	4	4	4	4	4
48	4	4	4	4	4	4	4
49	4	4	4	4	4	4	4
50	4	4	4	4	4	4	4
51	4	4	4	4	4	4	4
52	4	4	4	4	4	4	4
53	4	4	4	4	4	4	4
54	4	4	4	4	4	4	4
55	2.5	2.5	2.5	2.5	2.5	2.5	2.5
56	2.5	2.5	2.5	2.5	2.5	2.5	2.5
57	2.5	2.5	2.5	2.5	2.5	2.5	2.5
58	4	4	4	4	4	4	4
59	4	4	4	4	4	4	4
60	4	4	4	4	4	4	4
61	4	4	4	4	4	4	4
62	4	4	4	4	4	4	4
63	4	4	4	4	4	4	4
64	4	4	4	4	4	4	4
65	4	4	4	4	4	4	4
66	2.5	2.5	2.5	2.5	2.5	2.5	2.5
67	2.5	2.5	2.5	2.5	2.5	2.5	2.5
68	2.5	2.5	2.5	2.5	2.5	2.5	2.5
69	2.5	2.5	2.5	2.5	2.5	2.5	2.5
70	2.5	2.5	2.5	2.5	2.5	2.5	2.5
71	2.5	2.5	2.5	2.5	2.5	2.5	2.5
72	2.5	2.5	2.5	2.5	2.5	2.5	2.5

Appendix 4 – Exogenous Land and Building cost parameters (ref. p. 28)

Capital Cost per square foot equation parameters and variable inputs:

Capital cost parameter by land class:

parameter	value
ind cost	38.038
com cost	39.611
res cost	39.611
ind min	50
com min	50
res min	50

FAR class parameters:

FAR class	Min FAR	Avg FAR	Max FAR
1	0	0.2	0.25
2	0.25	0.375	0.5
3	0.5	0.625	0.75
4	0.75	0.875	1
5	1	1.25	1.5
6	1.5	1.75	2
7	2	3.5	5
8	5	7.5	15

(note: the base capital cost per square foot equation uses only “AvgFAR” variable in the table above; the min and max terms are shown only for illustrative purposes.)

Base Capital cost per square foot by land class:

ezone	ind	com	res		ezone	ind	com	res
1	103.43	168.26	168.26		37	77.5725	126.195	126.195
2	93.087	151.434	151.434		38	77.5725	126.195	126.195
3	93.087	151.434	151.434		39	82.744	134.608	134.608
4	82.744	134.608	134.608		40	82.744	134.608	134.608
5	93.087	151.434	151.434		41	72.401	117.782	117.782
6	93.087	151.434	151.434		42	72.401	117.782	117.782
7	82.744	134.608	134.608		43	77.5725	126.195	126.195
8	82.744	134.608	134.608		44	77.5725	126.195	126.195
9	82.744	134.608	134.608		45	77.5725	126.195	126.195
10	82.744	134.608	134.608		46	82.744	134.608	134.608
11	82.744	134.608	134.608		47	82.744	134.608	134.608
12	77.5725	126.195	126.195		48	77.5725	126.195	126.195
13	77.5725	126.195	126.195		49	77.5725	126.195	126.195
14	82.744	134.608	134.608		50	77.5725	126.195	126.195
15	82.744	134.608	134.608		51	77.5725	126.195	126.195
16	82.744	134.608	134.608		52	77.5725	126.195	126.195
17	82.744	134.608	134.608		53	77.5725	126.195	126.195
18	82.744	134.608	134.608		54	77.5725	126.195	126.195
19	77.5725	126.195	126.195		55	72.401	117.782	117.782
20	77.5725	126.195	126.195		56	72.401	117.782	117.782
21	77.5725	126.195	126.195		57	72.401	117.782	117.782
22	77.5725	126.195	126.195		58	82.744	134.608	134.608
23	77.5725	126.195	126.195		59	82.744	134.608	134.608
24	77.5725	126.195	126.195		60	77.5725	126.195	126.195
25	72.401	117.782	117.782		61	77.5725	126.195	126.195
26	77.5725	126.195	126.195		62	77.5725	126.195	126.195
27	72.401	117.782	117.782		63	77.5725	126.195	126.195
28	77.5725	126.195	126.195		64	77.5725	126.195	126.195
29	77.5725	126.195	126.195		65	77.5725	126.195	126.195
30	77.5725	126.195	126.195		66	72.401	117.782	117.782
31	77.5725	126.195	126.195		67	72.401	117.782	117.782
32	77.5725	126.195	126.195		68	72.401	117.782	117.782
33	77.5725	126.195	126.195		69	72.401	117.782	117.782
34	77.5725	126.195	126.195		70	72.401	117.782	117.782
35	72.401	117.782	117.782		71	72.401	117.782	117.782
36	72.401	117.782	117.782		72	72.401	117.782	117.782

Land Cost per square foot equation parameters and variable inputs:

Land cost parameter by land class:

parameter	value
ind cost	7.4942
com cost	16.551
res cost	13.758

Base land cost per square foot by land class:

ezone	ind	com	res	ezone	ind	com	res
1	24.45	46.98	39.78	37	1.2225	2.385919	1.989
2	18.3108	23.60763	22.86225	38	2.378044	4.832017	4.330838
3	15.8925	14.20959	15.90649	39	4.261844	11.39786	5.922774
4	6.1125	8.593883	8.689988	40	0.846747	5.097321	3.413889
5	7.216084	17.06917	14.55289	41	2.776763	3.908391	4.821906
6	10.49874	11.20073	10.93757	42	1.555952	4.046685	2.9835
7	1.789902	11.84705	11.2347	43	3.142197	6.944387	5.967
8	4.89	9.394919	10.55739	44	4.183029	6.506976	7.176544
9	6.922173	7.673163	10.22428	45	3.671005	6.309666	7.090812
10	6.430773	7.330173	6.390072	46	3.56831	7.167792	6.430034
11	3.282067	6.941578	6.237787	47	6.289945	6.371994	13.77358
12	2.927414	11.745	4.9725	48	4.246381	7.203667	6.475516
13	2.713739	6.621481	8.782273	49	2.437556	4.962017	9.676288
14	4.503351	8.117158	7.490371	50	3.879647	6.965956	12.48868
15	3.995634	7.860863	7.40978	51	3.363381	5.154654	12.40767
16	2.125374	6.650954	5.984294	52	3.30075	6.329729	11.5855
17	4.057838	7.643992	7.435319	53	2.537407	5.601815	9.92937
18	6.261433	7.127082	6.068338	54	4.356572	5.319485	9.766865
19	2.650328	5.200661	4.542645	55	1.962921	4.355086	5.098071
20	4.89	6.925821	5.740785	56	2.225774	4.500677	11.03771
21	2.2031	5.968639	5.230845	57	2.112768	3.291033	5.000915
22	2.03608	5.302691	5.365182	58	6.289945	6.371994	13.77358
23	1.854977	5.949221	7.204865	59	6.289945	6.371994	13.77358
24	1.604894	5.91455	7.316307	60	2.713739	6.621481	8.782273
25	1.2225	0.880659	0.598625	61	2.225774	4.500677	11.03771
26	5.847264	13.81689	21.52613	62	2.112768	3.291033	5.000915
27	0.12225	0.2349	0.1989	63	3.671005	6.309666	7.090812
28	2.759783	6.124649	5.128703	64	2.378044	4.832017	4.330838
29	3.05625	5.455021	7.356242	65	2.225774	4.698	3.978
30	3.247394	4.471456	6.463923	66	4.183029	4.698	3.978
31	2.830311	5.030128	2.619404	67	2.112768	4.698	3.978
32	2.288338	3.908573	5.721531	68	1.555952	4.698	3.978
33	2.506613	3.671817	4.213073	69	1.2225	4.698	3.978
34	1.469787	2.056052	0.730318	70	1.467	4.698	3.978
35	1.782394	3.331572	3.56999	71	1.2225	4.698	3.978
36	1.279824	2.941988	1.989	72	1.2225	4.698	3.978

Minimum land cost per square foot by land class:

ezone	ind	com	res		ezone	ind	com	res
1	4	4	4		37	2.5	2.5	2.5
2	4	4	4		38	4	4	4
3	4	4	4		39	4	4	4
4	4	4	4		40	4	4	4
5	4	4	4		41	2.5	2.5	2.5
6	4	4	4		42	2.5	2.5	2.5
7	4	4	4		43	4	4	4
8	4	4	4		44	4	4	4
9	4	4	4		45	4	4	4
10	4	4	4		46	4	4	4
11	4	4	4		47	4	4	4
12	4	4	4		48	4	4	4
13	4	4	4		49	4	4	4
14	4	4	4		50	4	4	4
15	4	4	4		51	4	4	4
16	4	4	4		52	4	4	4
17	4	4	4		53	4	4	4
18	4	4	4		54	4	4	4
19	2.5	2.5	2.5		55	2.5	2.5	2.5
20	2.5	2.5	2.5		56	2.5	2.5	2.5
21	2.5	2.5	2.5		57	2.5	2.5	2.5
22	2.5	2.5	2.5		58	4	4	4
23	2.5	2.5	2.5		59	4	4	4
24	2.5	2.5	2.5		60	4	4	4
25	2.5	2.5	2.5		61	4	4	4
26	4	4	4		62	4	4	4
27	2.5	2.5	2.5		63	4	4	4
28	4	4	4		64	4	4	4
29	4	4	4		65	4	4	4
30	4	4	4		66	2.5	2.5	2.5
31	4	4	4		67	2.5	2.5	2.5
32	4	4	4		68	2.5	2.5	2.5
33	4	4	4		69	2.5	2.5	2.5
34	2.5	2.5	2.5		70	2.5	2.5	2.5
35	2.5	2.5	2.5		71	2.5	2.5	2.5
36	2.5	2.5	2.5		72	2.5	2.5	2.5