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**METROSCOPE 3.0 GENERALIZED DESIGN AND EQUATION
SPECIFICATIONS AS IMPLEMENTED AT PORTLAND METRO JULY 1,
2007**

Introduction

The following report comes in two basic sections. The first section deals with the description and documentation of MetroScope in an exclusively verbal or schematic format. We describe the model's operation in terms of decision-making modules. We define what is being decided on and we elaborate on the arguments that go into each of the decisions. This first section provides a general understanding of the logic and rationale of the MetroScope modeling procedure. The intent of the section is that it be accessible to any motivated reader without the burdensome necessity of deciphering substantial tracts of mathematical notation.

The second section provides the equation details. The second section first includes a verbal summary of the equation structure – equation by equation without delving into specific mathematical notation. We then provide an explicit mathematical representation of the model simulation system as it is presently used. Those versed in mathematical notation may find the second approach less confusing. Readers with a technical bent but put off by mathematical notation may find the first option useable.¹

SECTION ONE

Urban Simulation Models – Explanation and Some History

MetroScope belongs to a class of models that are generically called “urban simulation models”. What these models purport to do is to estimate current and future values of one or more measurable urban attributes. Certainly, the most common and well known of these urban simulation models is the 4 step transportation model. This model developed in the 1950's from even earlier hand and calculator based traffic engineering approaches measures, estimates and forecasts the attributes of urban traffic. Originally, the transportation model sought to estimate total auto trips over a large geography. This information converted into “desire lines” could then be interpreted by experienced traffic engineers and used to determine the timing and size of future regional roads. Over a 50 year period, the transportation model has seen numerous improvements so that now the model estimates and forecasts detailed trip data for a variety of trip modes, purposes, times, and destinations at a block level geography over highly detailed transportation networks.

Growing use of the transportation model during the late 50's and early 60's underscored the need for similar methods for estimating the attributes of land use; particularly population and employment. Growing use of computers, data bases and the application of neoclassical economics to problems of urban location quickly lead to proposals to combine transportation and land use models into “Integrated Transportation and Land Use” models wherein many measurable

¹ I personally find the verbal section very difficult and confusing; giving ample evidence as to why civilization has developed several mathematical notations over the course of history.

attributes of urban areas were to be estimated and forecast in one model. These proposals saw the development of 3 models in the 1960's that purported to be "Integrated Transportation and Land Use" models. These models were one: the Herbert – Stevens Model, two: EMPIRIC and three: the Lowry Spatial Interaction Model.

Of the three models only the Herbert – Stevens Model² formally embodied micro-economic theory and explicit market clearing. Unfortunately, it relied on very incomplete and simplified housing demand and supply equations and reflected a very poor knowledge of urban real estate. Limited computer capacity and over reliance on the linear programming techniques of the early 60's hampered development of the model and finally lead to its abandonment.

EMPIRIC³ represented an attempt to simulate urban development in a purely econometric format with numerous statistical equations estimated from cross –sectional data for urban areas. The impetus for EMPIRIC was the success of the national accounting models that simulated the macro measures of the economy such as GNP, national income and employment growth. EMPIRIC quickly came to grief when the estimated equations combined into a system failed to produce outputs on urban accounts that made any sense. Failure to have an explicit urban accounting structure similar to the National Income and Product Accounts (NIPA) provided a classic early example of what came to be known famously as the GIGO (garbage in, garbage out) principle. EMPIRIC too, was abandoned without implementation.

The Lowry Spatial Interaction Model⁴ was an adaptation of the "law of social gravity" introduced into the literature by J.Q. Stewart in a 1947 *Geographical Review* paper⁵ and later elaborated by G.K.Zipf⁶ and several other authors. The Lowry model emphasized the link between distance or travel time to work and residential location. It also recognized the link between certain types of employment and household location. Given an external estimate of "basic" employment both households and non basic employment could be located as some function of distance. Neither supply, nor prices, nor markets, entered into the model. The strength of the Lowry model was that it fit cross sectional data well; could be easily estimated and required relatively little data to calibrate and operate. The weakness of the model was that it only responded to changes in travel time; consequently leaving out most of the information relevant to urban development. However, of the 3 model types; the Lowry model and its descendants were implemented and continue to be used in some MPO's to the present day.

Where does MetroScope fit into the above integrated model typology? In terms of structure MetroScope is most like the Herbert-Steven's model. It does harbor within its multitudinous equations a fairly straightforward neoclassical demand and supply structure with a requirement that we find a price for each location and real estate type that matches demand and supply. However, in terms of how the equations are developed and their role in the model, MetroScope borrows heavily from the other model types as well. For instance, the Herbert-Steven's model did not use statistical equations for preference and choice; nor did it account for

² J. D. Herbert, B.H. Stevens, *A Model for the Distribution of Residential Activity in Urban Areas*, **Journal of Regional Science**, 2. (February 1960), pp. 21 – 36.

³ D.M. Hill, D. Brand, W. B. Hansen, *Prototype Development of a Statistical Land Use Prediction Model for the Greater Boston Region*, **Highway Research Record**, No 114, (1965), pp. 51 – 70.

⁴ I.S. Lowry, *A Model for Metropolis*, The Rand Corporation RM-4033-RC, (1964).

⁵ J.Q. Stewart, *Empirical Mathematical Rules Concerning the Distribution and Equilibrium of Population*, **Geographical Review**, XXXVII, July 1947, pp. 461 – 485.

⁶ G.K. Zipf, *The Hypothesis of the Minimum Equation at Unifying Social Principle*, **American Sociological Review**, XXII, December 1947, pp 627 –650.

such factors as neighborhood quality; nor the tradeoff between travel time to work and housing price. Using statistical equations more in the spirit of the Lowry Model and to a much less extent EMPIRIC, MetroScope accounts for all the obvious preference factors in real estate choice plus “averages” the responses in recognition of factors and circumstances that cannot be accounted for.

If you have little or no interest in the intellectual history of urban simulation models, you are probably wondering where we are going with this obvious digression from a MetroScope description. Our destination here is to point out a blatant and often overlooked (even by planning academics) difference between MetroScope type models and many statistical models in use today. For many models fitting of statistical equations to observed data constitutes almost all the work of model building. Once that chore is completed you plug in the independent variables (exogenous assumptions) and make a forecast. Only one iteration per time period of the model is required. All the outputs from the model equations are assumed to be consistent with one another. Commonly econometric models of the national and regional economies work that way as do urban simulation models in the Lowry or EMPIRIC tradition. Models in the Herbert-Steven’s tradition ⁷do not.

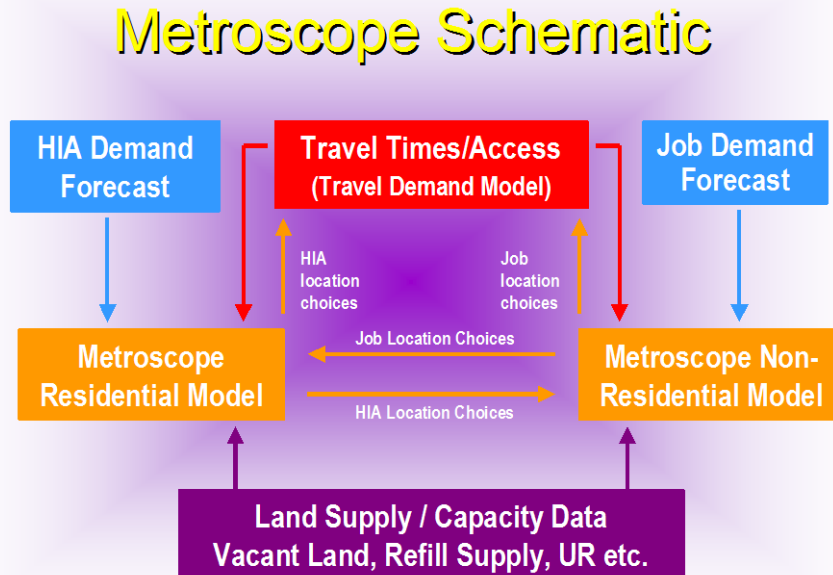
In MetroScope (as in most such models) all the demand and supply equations in addition to whatever other variables are included also include the price variable by location and real estate type. Demand responds negatively to an increase in price and supply responds positively to an increase in price. Consequently, supply and demand do not automatically match one another. The model must adjust prices iteratively for supply and demand to match for all locations and real estate types and the market to clear. In MetroScope statistical fitting of equations constitutes about 20 – 25% of the work. Establishing the equation structure, calibration to base year initial conditions and insuring the model iterates to a stable, consistent equilibrium in each forecast period constitute most of the MetroScope development effort.

Schematic Approaches to a Verbal Understanding of MetroScope

In the verbal description section we are following the convention of providing an overall view of MetroScope by using a schematic that portrays the basic computational modules as boxes and uses arrows to denote information flow between the modules. Figure A below shows the most general schematic.

⁷ MEPLAN, TRANUS, PECAS, MUSSA, Metrosim, and MetroScope come to mind as models that require solution (some type of iterative procedure) for prices that achieve market clearing in each time period. Papers on these models may be obtained at: 2nd Oregon Symposium on Integrating Land Use and Transport Models, Portland, OR., July 2000 (CD ROM: Oregon DOT, William J. Upton: william.j.upton@odot.state.or.us). We should also point out that the travel demand model is also an iterative, equilibrium model in that origin-destination travel times are required to be consistent across the generation, mode choice, distribution and network assignment modules.

FIGURE A: METROSCOPE GENERAL SCHEMATIC



Blue Boxes

Following the schematic shown above - the information contained in the blue boxes, HIA (household size, income, age, etc) demand and job demand, are provided as regional control totals every 5 years and are forecast by Metro’s regional econometric model. On the HIA (demographic) side we presently have regional estimates for every 5 year forecast period for 5 classes of household size, 5 classes of age of household head, 8 classes of household income and 2 classes of school age children present (yes or no). Combinatorics being what they are these classes taken one at a time yield 400 HIA classes which the model need process through the residential demand module in each 5 year period.⁸ On the employment demand side the numbers are much more modest as we presently use 15 employment classes (ie agriculture-mining, retail trade, medical-social service, etc.). Important here is that these variables are supplied at the regional level by the econometric model and MetroScope then finds a location within the region for them.

Purple Box

The other source of external information that MetroScope requires is located in the purple box at the bottom of the schematic. The data that enter the model here at each 5 year forecast interval are of two varieties. The first variety amounts to the basic initial land use conditions of the region being modeled. These data include the amount of vacant land in each location by zoning class that is or will be available for building during the forecast period. The data also include the land already considered developed that will be available during the forecast period

⁸ Actually there are 360 effective classes, since 1 person household with children present is logically precluded from occurring though the 2000 Census lists about 65 such households (legally emancipated children).

through infill and redevelopment. In addition, the model recognizes urban renewal efforts that produce additional resources and building that would not otherwise appear in a market based model.

Beyond these initial period accounts MetroScope land and policy inputs are updated every 5 years to reflect Urban Growth Boundary expansions and the creation (deletion) of new (existing) urban renewal districts. The model inputs also provide for zone changes, changes in development charges (SDC's) and changes in subsidy levels. Once land data are entered into the model, MetroScope automatically accounts for land consumption by location, zoning and land source (ie vacant, infill and redevelopment, urban renewal, UGB add and rural).

Red Box

The red box contains the travel demand model. The travel demand model consists of 4 different modules. These are (1) the trip generation module, (2) the mode split module, (3) the distribution module and (4) the network assignment module⁹. As presently implemented the travel demand modules use HIA data by TAZ (census tract in this case) that is collapsed into 64 classes and employment data by TAZ that is collapsed into 3 classes. Output data from the transportation demand model are travel times or logsums between all TAZ pairs. These data are then used in both the residential and nonresidential real estate modules as one of the variables used to determine demand for location in particular zones. Information is exchanged during each "major iteration" between the real estate modules and the travel demand modules of MetroScope within a forecast period. Since the travel demand model is well documented elsewhere, we do not describe this model in any more detail within this report.

Brown Box 1 – Residential Real Estate Model

Since both the econometric model and travel demand model have been well documented elsewhere, the two brown boxes delimiting the residential and nonresidential real estate models comprise the focus of the present documentation. Glossing over computation details at this point the residential model has the following major components.

Demand Module -

- Assign households by HIAK class and place of employment to owner or renter.
 - Assign households by HIAK class, place of employment, and owner/renter to 8 housing consumption bins.
 - Assign households by HIAK class, place of employment, owner/rent, and housing consumption bin to single family or multi-family housing type.
 - Assign households by HIAK class, place of employment, owner/rent, housing consumption bin, and housing type to one of 425 location choices.
- Compute for households by HIAK class, place of employment, owner/rent, housing consumption bin, housing type, and location how much they are paying for their housing.

⁹ The traffic assignment module uses proprietary software licensed by PTV America and is not subject to the open source license. MPO's using MetroScope should be prepared to provide their own network assignment module and whatever coding would be required (usually very little) to convert distribution module output into a form useable for their particular assignment module.

Supply Module –

- Determine for each of 425 locations (R zones corresponding to census tracts) the amount of residential capacity available in a forecast period by zoning class.
- Determine for each of 425 locations, consumption bin and zone class the cost of producing a house.
- Determine for each location consumption bin and zone class whether the demand price is high enough to support production (ie exceeds production cost).
- For all feasible production determine supply output by location, consumption bin and zone class and compute land consumption by location and zone class.
- Subtract land used by location and zone class from available supply and determine total land supply available for next period.
- Add new production to vintage (existing) housing by location and consumption bin for each location to determine total supply available.

Equilibrium Iteration –

- For each location zone determine total supply and total demand
- For zones with excess supply reduce the “location price” by a specified increment. For zones with excess demand increase the “location price” by a specified increment.
- Go to next iteration until maximum number of iterations is reached.¹⁰
- Pass the total households summed by employment zone (groups of R zones) to the nonresidential model.
- When the residential model and nonresidential models have exchanged data a set number of times, stop the run and pass the HIAK data by location (R Zone – TAZ) to the travel demand model.

Brown Box 2 – Nonresidential Real Estate Model

The nonresidential model works in much the same manner as the residential real estate model. As presently configured the nonresidential model uses fewer zones than the residential model – 72 E zones as compared to 425 R zones. This does not reflect a limitation in computational ability of the software and hardware. Rather it reflects a realistic appreciation of the heterogeneity of nonresidential real estate and the resultant very high level of data error engendered from small zone sizes. The major modules of the nonresidential model we list below. For the nonresidential model we presently use 15 employment categories that we assign to 6 real estate types. These real estate types are manufacturing, warehousing, retail trade/services, general office, medical/social and government. Compared to the residential model the nonresidential model is much more aggregate and has far fewer moving parts and details. Aggregation and detail level is driven from the diversity of nonresidential uses, the large level of data error and an accompanying limitation of the level of knowledge about nonresidential real estate transactions on both the demand and supply sides. The good news is that this module runs very fast.

Nonresidential Demand

¹⁰ We use a set number of iterations rather than reducing sum squared error below a specified level. This insures that we always get the same result given identical inputs at the start of a run.

- Determine by SIC (NAICS), real estate type and location the demand for nonresidential real estate denominated in square feet.
- Determine by SIC (NAICS), real estate type and location the demand for employment denominated in BLS employees.

Nonresidential Supply

- Determine for 72 E zones the amount of nonresidential capacity available by zoning category (FAR class) by 3 land source classes of industrial, retail/general office and institutional.
- Determine for 72 E zones, land source and FAR class the cost per square foot of producing the 6 real estate types.
- Compare cost of construction to demand price for each of the 6 real estate types by E zone and FAR class to determine amount of square footage to be built in each E zone by real estate type.
- For each E zone and real estate type add the additional square footage to the vintage (existing) square footage to determine total supply available.

Equilibrium Iteration

- For each E zone and real estate type compare total supply square footage to total demand square footage.
- For E zones and real estate types with excess supply decrease the location price a specified amount. For E zones and real estate types with excess demand increase the location price a specified amount.
- Go to next iteration, stop when the maximum number of iterations is reached.
- Pass the total employment by Ezone to the residential model.
- When the nonresidential and residential models have exchanged household and employment data a set number of times, stop and pass the employment data by type to the travel demand model.

Outside the Box - Internal Iteration and Output

A couple of things are not explicit in the schematic. The first thing is the number of iterations between models in a given 5 year forecast period. All the models iterate internally for a set number of iterations or until certain stability conditions have been met. The real estate modules are set to iterate internally a set number of times (presently 25 iterations) and then exchange data with the other real estate model. Data exchange occurs 2 times after which the residential and nonresidential models are considered consistent with one another. Data from both models are sent to the travel demand model where trips are iterated through the distribution and assignment modules for a set number of iterations. These travel times then provide the travel times for the next iterations of the real estate models for the 5 year forecast period. The land use models are then run through their iteration sequence for one more cycle. When complete the real estate model outputs are sent to the travel model for a final set of iterations for the travel model and the outputs for the forecast period are considered complete. Since all models within MetroScope work on a set number of iterations, MetroScope has the property of producing identical outputs given identical inputs in repeated tests. **A small change in inputs or iteration**

number results in different output. Consequently, all policy and investment changes produce a measured effect. While perhaps old-fashioned, deterministic models do have an advantage over stochastic models¹¹ when you are trying to figure out what caused what.

Beyond the arrows pointing in various directions between boxes in the above schematic we make no explicit reference to output. MetroScope produces three types of output. These are:

- A very large number (100's of millions) of ephemeral outputs; which are overwritten in each iteration of the real estate and travel models. Some of these data are of interest for particular purposes and may be written to permanent files on a custom basis.
- Final iteration data that are used by other MetroScope modules and used in reports for each 5 year forecast period. These data are written out to a “data store” in PostgreSQL format and available for conversion to Access, Excel or other data base format.
- “Post processor” data that are created from the PostgreSQL files that provide special reports on redevelopment and infill, growth in centers, infrastructure cost, average commute distance, density, land use consumption by source, annual housing and transportation costs, etc.

In addition to tabular output we also have constructed a “MetroScopeViewer” that is a mapping program that directly reads the PostgreSQL outputs files and displays and compares one or two scenarios at a time.

SECTION TWO

Model Equation Details – In the Belly of the Beast

Before we start explaining operation of particular equations, it is helpful to list out the various zone, land use, demographic, zoning, etc. categories that are used in MetroScope. In the equations for purposes of readability we generally suppress as many subscripts and superscripts as possible. While increasing readability overall, lack of sub and superscripts can cause confusion about equation references and dimensionality. For this reason we list the various categories used in MetroScope and their count totals.

Demand Side Output Categories and Zones

Demographics – HIAK 400 classes

- H: Household size 5 classes from 1 – 4 persons with 5+ the top code class
- I: Income 8 classes in year 2000 census dollars with \$100,000 + being the top code class.
- A: Age of household head 5 classes with 65+ being the top code class.
- K: School age children present (5 – 18) in household 2 classes yes or no.

Geography – Rzones and Ezones

¹¹ While perhaps more like reality, stochastic models do have the property that you never get the same result twice (dependent on model complexity and/or resolution level). This makes error detection or policy analysis tedious for small errors or land use and investment policies implemented at a financially realistic level.

Rzones: 425 Rzones (year 2000 census tracts) comprising all of Clackamas, Clark, Columbia, Multnomah, Washington Counties and parts of Marion and Yamhill Counties.

Ezones: 72 Ezones comprising groups of census tracts (Rzones) covering same area as Rzones.

Residential real estate types – 4 types

- OSFD – Owner single family dwelling (detached unit)
- OMFD – Owner multi family dwelling (attached unit)
- RSFD – Renter single family dwelling (detached unit)
- RMFD – Renter multi family dwelling (attached unit)

Nonresidential real estate types – 6 types

- Manufacturing
- Warehousing/distribution
- Retail/consumer services
- General office/other commercial
- Medical/social/institutional (non government)
- Government

Employment classes - 15 SIC classes

- Agriculture – mining – forestry
- Construction
- Non durable manufacturing
- Durable manufacturing
- Electronic and other high tech manufacturing
- Transportation and warehousing
- Utilities and communications
- Wholesale trade
- Retail trade
- Consumer services
- Health/social services
- Business/professional services
- Other government
- K-12 education

Note that at this point we have 400 residential demand classes (HIAK) occupying 4 different residential real estate types in 425 Rzones. On the nonresidential side we have 15 employment classes occupying 6 different real estate types in 72 Ezones. All of the above categories exist on the demand side of the equation system. Residential HIAK and nonresidential Employment class are ephemeral in that they may inhabit different real estate types in any given 5 year period. By contrast both residential and nonresidential real estate types are semi fixed with regard to 5 year forecast period. To be clearer each real estate type in each 5 year forecast period “inherits” the vintage real estate from the previous 5 year period. Relevant residential and nonresidential

demand conditions in the forecast period determine whether additional real estate is added during the period.

For residential real estate on the demand side we use an additional category to describe each residential real estate type. This is the “consumption bin” of which there are 8 per real estate type. Each consumption bin corresponds to a “desired” housing consumption level in terms of house size and lot size. In general the lowest consumption bin (bin 1) is associated with very low income and a single person household while the highest consumption bin (bin 8) is associated with very high income and large household size. In each forecast period housing demand is tracked and assigned in terms of the 8 consumption bins. Likewise on the supply side the housing inventory is divided by housing type into 8 consumption classes that correspond to the demand classes.

Each consumption bin by the 4 housing types is first denominated into dwelling unit size and lot size. These data are then combined with Rzone specific information and coefficients from a hedonic equation to yield a demand price estimate for new housing in each Rzone for each of the 8 consumption bins. These calculations yield a demand price for each housing type by consumption bin which change by Rzone. The lot size and house size data are then passed to the supply side model that calculates the cost of construction by zone class by housing consumption bin by Rzone. The available housing capacity is used in a given Rzone for a given housing type in a given zoning class for those consumption bins where the demand price exceeds the construction cost.

Supply Side Zoning Categories

Moving to the supply side of MetroScope we have several layers of zoning, land source type and 2040 Plan design type to deal with. First off for each Rzone we have 34 zoning classes. These are:

0. Farm and Forest (FF)
1. Rural Residential Future Urban (RRFU)
2. Single Family 1 acre tax lot (SFR1)
3. Single Family ½ acre tax lot (SFR2)
4. Single Family 10000 sq. ft. lot (SFR3)
5. Single Family 9000 sq. ft. (SFR4)
6. Single Family 7000 sq. ft. (SFR5)
7. Single Family 6000 sq. ft. (SFR6)
- .
- .
- .
18. Multi-family –very low density (MFR1)
- .
- .
- .
24. Multi-family – very high density (MFR7)
25. Mixed-use commercial & residential (MUR1)

.
. .
34 Mixed-use commercial & residential (MUR10)

On the nonresidential side, for each Ezone MetroScope generalizes jurisdiction zoning into 3 basic classes. These are:

1. Industrial (allows the manufacturing and warehouse real estate types).
2. Commercial/ Office (allows the retail/services and general office types).
3. Institutional (allows the medical/social and government real estate types).

Furthermore, each nonresidential zoning class is broken into 8 floor to area classes (FAR). These classes are:

1. less than .25 acre (25% coverage)
2. .25 - .5 acre (25 – 50% coverage)
3. .5 - .75 acre (50 – 75% coverage)
4. .75 – 1 acre (75 – 100% coverage)
5. 1 – 1.5 acre (100 – 150% coverage)
6. 1.5 – 2.0 acre (150 – 200% coverage)
7. 2.0 – 5.0 acre (200 – 500% coverage)
8. greater than 5.0 acre (500 +% coverage)

Supply Side Land Source and 2040 Design Type Categories

For policy purposes MetroScope tracks land consumed and resultant real estate, demographic and employment allocations at a more detailed level than Rzone and Ezone divided into the categories listed above. All the categories listed above may be displayed by the following 5 land source categories.

1. Vacant land (Land officially shown as vacant in our land accounting system (RLIS)).
2. Refill land (Land that is officially shown as developed in our land accounting system but is expected to be redeveloped or developed more intensively (infill) in the future).
3. UGB adds (Land that is presently not in the Urban Growth Boundary but will be added at some point in the forecast period).
4. Urban renewal land (Land that is located within existing or assumed future urban renewal districts – urban renewal land differs from other land sources in 2 important ways. It need not be identified and justified at the parcel level and the production cost function for real estate can be subsidized (reduced) relative to other land within the Rzone or Ezone).
5. Neighboring jurisdictions and rural residential (Land that is located outside Metro boundaries in jurisdictions not subject to Metro or Clark County planning authority).

At the bottom of the land supply hierarchy we have 2040 Plan design types. These 2040 Plan design types fall into 3 broad classes and are designated as:

1. Centers – Includes land by Rzone and Ezone in all aforementioned supply categories that is designated as Central City, Regional Center or Town Center. Land is identified at the tax lot level.
2. Corridors – Includes land by Rzone and Ezone in all aforementioned supply categories that is designated as Corridor, Town Center and Station Area in the 2040 Plan. Land is identified at the tax lot level and must have zoning consistent with the Plan designation.
3. All Other Areas (AOA) – Primarily includes single family residential zoned land, industrial land, other employment areas and rural.

In sum then on the land supply side we track land consumption and real estate allocation over 4 dimensions. These are:

Rzone or Ezone
 Zoning class
 Land source
 Design type

The 4 fold land tracking system creates a substantial combinatoric explosion of output data. For instance for residential you have 425 Rzones x 34 Zoning classes x 5 Land sources x 3 design types equal to 216,750 possible combinations of land consumption for one housing type. Considering that we have 4 housing types the combinations increase to 867,000 possibilities.

For the residential supply module like the demand side we also track housing inventory and determine new production by 8 housing consumption bins. New production is given by Rzone and zone class that by definition can be further subdivided into land source and design type. Housing consumption bin categories by Rzone and house type have counterparts on the demand side as well so on a pro rata basis it is possible to link HIA occupancy to the above classification scheme.

MetroScope Real Estate Model Equation System – Details in Narrative Form

Up to this point we have greatly generalized the actual arithmetic statements (algorithms) that constitute the computational guts of MetroScope. In this section we shall actually walk through a narrative account of each of these statements. However, in both this section and in the next section we suppress redundant detail that in reality the computer handles over and over again. For example, for every E zone and HIAK class (72 x 400) the computer must evaluate the arguments of the same housing tenure choice equation. Obviously, since only the value of the arguments change, we repeat it here only once; not 28,800 times. Arguments taken as exogenous at the regional or zone level are given in red; arguments determined within the model (endogenous) are denoted in blue; arguments exogenous in the model being described but

endogenous from another MetroScope model are denoted in green¹². Lagged endogenous variables are denoted in brown.

Residential Model – Equations in English (Almost) Demand Module:

In order to better understand the equation descriptions we first discuss the various classes and zone systems that are employed in MetroScope. The present discussion we limit to residential. Nonresidential classes and zone systems we discuss in the nonresidential section except where they are pertinent to the residential model. The MetroScope residential model recognizes 4 classes of real estate. These are OSFD – owner occupied single family, OMFD – owner occupied multi-family, RSFD – renter occupied single family and RMFD – renter occupied multi-family. While owner and renter are self-evident classifications, single family and multi-family require explanation. Single family is best understood as a dwelling unit that you can walk around without walking around another person’s house. Multi-family is everything else. This means that row houses and the mysterious US Census category of “single family 2 – 3 units attached” goes into the multi-family category.

The residential model uses a 425 Rzone (for residential zone) system that consists of the year 2000 census tracts that comprise MetroScope’s 6 + county coverage area. There are no limitations on the number of zones save computation time, volume of output and the increasing level of data error associated with ever smaller zones. We find that post processing census tract based output to smaller TAZs works very satisfactorily and provides an opportunity to use unique GIS data to provide details “to the block” allocations should one choose. Our experience points to the wisdom of starting with fewer zones and adding more as users gain experience with MetroScope.

The MetroScope residential model also uses the 72 Ezones of the nonresidential model. These are groups of year 2000 census tracts. The logic of MetroScope requires that each household have a primary place of work or “service centroid” in the case of non-worker households.¹³ From each of the 72 Ezones, households than select a place of residence from among the 425 Rzones according to rules embedded in the arguments of the indirect utility functions we have estimated and calibrated for the model. This procedure yields an interpretable “bid rent” function that allows computation of willingness to pay and consumer surplus. It also provides an explicit linkage between housing stock and place of employment. Thus we are able to calculate associated public welfare measures such as average travel distance and consumer surplus associated with any particular policy initiative.

¹² For instance travel times are exogenous to both the residential and nonresidential real estate models but are produced endogenously in the travel demand model. Similarly the number of households supported from a particular employment zone is exogenous to the residential model but endogenous in the nonresidential model.

¹³ Some households may be temporarily unemployed or older households retired. In these cases we presume that the last place of work serves as the service centroid. Avoiding further elaboration we assume that the probability of such events is proportional to total Ezone employment.

The final salient aspect of the zone system is that it establishes the number of simultaneous equations in both the residential and nonresidential models that must be solved for price. The formula for number of equations and unknowns (location price by real estate type and zone) is the number of zones times the number of real estate types. For the residential models as used in MetroScope this amounts to 425 zones times 4 housing types equals 1,700 equations that yield 1,700 location prices in each 5 year period.

Weighted Price and Frequency Accounting Identities:

As reported several times already, MetroScope equations communicate with one another via the mechanism of price as any market based model must do. Consequently, all independent equations¹⁴ within MetroScope contain some measure of real estate price on the right hand side of the equation. Rather than digress to explain the construction of a price variable when it comes up in an equation, we cover prices and a related set of relative frequency variables at the start. To help with understanding we denote the accounting identity equations with the term “AI” while the independent behavioral equations are counted as 1,2,3, etc.

Base Level Price Generation Procedure Rzone Based:

Consider that for a given time period for a specific iteration for a specific Rzone and housing type (OSFD, OMFD, RSFD and RMFD), MetroScope has determined a location price. The location price is then converted into a price index by Rzone by housing type via the accounting identities:

- R 1.** RSFD Location Price in Rzone at forecast time period one = [Rzone price index calibration year for renter single family] times [Rzone location price adjustment factor for renter single family]
- R 2.** RMFD Location Price in Rzone at forecast time period one = [Rzone price index calibration year for renter multi family] times [Rzone location price adjustment factor for renter multi family]
- R 3.** OSFD Location Price in Rzone at forecast time period one = [Rzone price index calibration year for owner single family] times [Rzone location price adjustment factor for owner single family]
- R 4.** OMFD Location Price in Rzone at forecast time period one = [Rzone price index calibration year for owner multi family] times [Rzone location price adjustment factor for owner multi family]
- R 5.** Rzone Supply Price by House Type = (Current Period real estate price index by Rzone and House Type/ Base year Calibration Price index by Rzone and House Type)

¹⁴ Accounting and procedural equations are not independent and may or may not include price variables.

Equation R 5 is repeated for each housing type (RSFD, RMFD, OSFD, OMFD). Basically, the supply price calculation provides an index of how much a particular housing type in any given Rzone has changed since the calibration base year.

At this point in our explanation we make the simplifying procedure of using only one tenure and house type wherever possible since the same procedures are repeated for each tenure and house type.

Housing Price Weighting Procedures Rzone Based:

R 6. **Weighted owner price by Rzone** = {[Rzone OMFD index] times [Rzone OMFD supply] + [Rzone OSFD index] times [Rzone OSFD supply]} divided by [Rzone OMFD supply + Rzone OSFD supply]

The above equation yields a weighted owner price (OSFD and OMFD) by Rzone. Identical procedures yield a weighted renter price (RSFD and RMFD) by Rzone and an overall weighted Rzone price by all tenures and types.

Note: For our right hand side endogenous variables we calculate weighted indices using Rzone supply totals lagged one forecast period. While feasible to use current period supply data, it does not improve model performance and increases model run times as big Rzone supply oscillations perturb price indices¹⁵.

Housing Price Weighting Procedures Ezone Based:

For purposes of determining tenure choice and housing type choice of households supported from each Ezone, we calculate a set of Ezone specific weighted price indices.

Unlike the Rzone specific indices, these are aggregate indices containing information from all 425 Rzones evaluated from the perspective of each Ezone. This procedure recognizes the fact that someone employed in the CBD faces a different set of tenure and housing type choices than does an equivalent household employed on the urban periphery.

To create the Ezone specific rent, own, and house type price indices we use a similar procedure as above but divide the Rzone supply by the travel travel time from the Ezone being evaluated to each Rzone. Hence we have:

R 7. **Weighted Owner SFD price index by Ezone** = {Sum over all Rzones from each Ezone [Rzone OSFD price index] times {[Rzone OSFD supply] divided by [travel time between Ezone and Rzone]}} divided by {Sum over all Rzones from each Ezone {[Rzone OSFD supply] divided by [travel time between Ezone and Rzone]}}

¹⁵ The supply schedules as may be apparent in the supply equations are discrete, stepped functions unlike the demand functions which are continuous. This means that for Rzones with very large capacity relative to the existing base of a homogenously zoned housing product very small changes in housing price may increase or decrease housing supply in that Rzone by a very large amount. Passing these supply oscillations through to other endogenous variables greatly increases model run time.

Repeating the procedure for the other three housing types yields the appropriate weighted Ezone price index for each of them. In this manner the housing price aggregate variable changes by Ezone even though data from all 425 Rzones is being used for each Ezone.

Similarly, we construct weighted owner price index by Ezone as follows:

R 8.
$$\text{Weighted Owner price index by Ezone} = \frac{\{[\text{Ezone weighted Owner MFD price index}] \times [\text{Ezone travel time weighted Owner MFD supply}] + [\text{Ezone weighted Owner SFD price index}] \times [\text{Ezone travel time weighted Owner SFD supply}]\}}{\{[\text{Ezone travel time weighted owner MFD supply}] + [\text{Ezone travel time weighted owner SFD supply}]\}}$$

As above repeating the same process yields the Ezone weighted renter price index and the overall Ezone price index.

Despite the above labyrinth of language the above scheme is simple. We are starting with a Rzone based price index specific to housing and tenure type. We then convert the Rzone indices to an Ezone perspective by weighting each Rzone index by its housing supply divided by the travel time between the Ezone being operated upon and the Rzone.

Housing Frequency, Auto Cost and Transit Cost by Tenure and Type Weighting Procedure Ezone Based:

We also compute from an Ezone perspective a housing frequency variable and price indices for auto cost and transit cost. The auto cost variable is the complement of owner choice and single family. Transit cost is the complement of renter choice and multi-family. All 3 variables are computed from the perspective of each Ezone.

R 9.
$$\text{Tenure (Own) Frequency by Ezone} = \frac{\{[\text{sum over all Rzones } [\text{R Zone supply of OSFD} + \text{Rzone supply of OMFD}]] \text{ divided by } [\text{travel time from Ezone to Rzone}]\}}{\{[\text{sum over all Rzones } [\text{Rzone supply of OSFD} + \text{Rzone supply of OMFD}]] \text{ divided by } [\text{travel time from Ezone to Rzone}]] + [\text{Rzone supply of RSFD} + \text{Rzone supply of RMFD}] \text{ divided by } [\text{travel time from Ezone to Rzone}]\}}$$

The same procedure is applied to produce the housing frequency by Ezone for renter tenure. Housing type frequency by Ezone is calculated as shown below.

R 10.
$$\text{Housing Type Frequency by Ezone} = \frac{\{[\text{sum over all Rzones } [\text{R Zone supply of OSFD}]] \text{ divided by } [\text{travel time Ezone to Rzone}]\}}$$

For Housing Types other than OSFD the equations are identical except that OMFD, RSFD and RMFD values are substituted in the appropriate equation.

In MetroScope 3.0 for the residential model we have incorporated arguments for the transportation complements of single family and multi family housing. National Survey of Consumer Expenditure data and some local data suggest that auto composite cost (index of travel time and \$ cost) is the complement of single-family consumption and that transit (non-auto alone) composite cost is the complement of multi-family consumption. At this point we are unable to obtain satisfactory parameters for these terms so the values are set at one in the present model configuration. As a point of information when computed from the Ezone perspective the auto and transit price indices are weighted by single family Rzone totals divided by travel time and multi-family Rzone totals divided by travel time, respectively.

At this point we need reiterate and clarify the calculation and perspective of all internally calculated residential indices. MetroScope has two perspectives when calculating price indices – Rzone and Ezone. The Rzone perspective is simple: it is whatever the price index in that Rzone is for a household located inside it. The Ezone perspective is more complicated: this is a measure of all Rzone values of a variable when viewed from the perspective of someone employed within the Ezone. In this case, the contribution of each Rzone value to the weighted index is divided through by the travel time from the particular Ezone to the Rzone. In this manner we acknowledge that nearby prices and product frequencies are of more importance than those at a greater distance. As noted in the equation descriptions these weighted indices are used in the tenure choice and housing type equations. They are not used in the location choice equations.

Rzone Bin Frequency (Odds Ratio)

Assigning households to one of eight “consumption bins” is a method of overcoming the bane of housing price elasticity and housing location choice models; namely the spatial mismatch between a household’s desired housing consumption level and the availability of choices in particular locations. Housing by size, age, quality and neighborhood value is not randomly and continuously distributed. For reasons of history, topography, business cycles and chance, housing is very heterogeneous in space. In plainer terms we do not expect someone shopping for \$200,000 sports cars to go to Dodge Neon showrooms; nor someone shopping for \$10,000 sedans to patronize Ferrari dealerships. In sum, the model housing decision structure must reflect the observation that people only consider locations with products that are relevant to their choice. Assigning households to “consumption bins” and restricting their choices to housing belonging to the same bin satisfies the relevancy of choice concept¹⁶.

The numerator of Equation 18 (below), the housing location choice equation, less the Rzone bin frequency argument, can be interpreted as the utility of one housing unit located in a particular Rzone from the perspective of a household working in a particular Ezone. This utility estimate is then weighted by the number of housing units of the appropriate consumption bin actually located

¹⁶ Results from poorly identified statistical analyses have led to some misguided presumptions in the planning literature such as “high income households do not prefer living close to the CBD”. Actually, due to history and development many areas have little housing or neighborhoods attractive to high income households close to the CBD. Success of the Pearl District, the West End and South Waterfront indicate exactly the opposite of the guidelines taken from statistical work.

in the Rzone. **Households only consider Rzones with appropriate housing stock; they do not consider irrelevant choices.**

Rzone bin frequency is defined in the procedure outlined below.

R 11. **Rzone Housing Frequency by Housing Type and Bin** = [number of units in Rzone in particular consumption bin by housing type] divided by [regional average number of housing units in consumption bin by housing type.]

The values referenced above are calculated at the start of each forecast period as follows:

1. Divide regional housing stock by tenure and type (OSFD, OMFD, RSFD, RMFD) into eight equal in number consumption/bin – value categories based on year 2000 Census value and calculate the regional average for each census tract of the housing stock by housing type and consumption bin.
2. Based on regional level census value break points assign housing stock by type in each Rzone to the eight consumption/bin – value categories.
3. Make the calculation as noted in equation **R 11**.

Also keep in mind that we use the Rzone bin frequency weighting argument as a lagged variable so the total “attractiveness” of a given Rzone depends on the bin frequency weighting factor from the previous 5 year period. While it is possible to calculate this argument on a current endogenous level, it does not improve model operation and increases model run time. This procedure eliminates the possibility of oscillations and need only be done at the start of any forecast run period.

Construction of Exogenous Variables

Up to this point we have taken the exogenous arguments as “givens”. However, this would be misleading. In the MetroScope residential demand module we recognize 2 types of exogenous variables: the year 2000 observed initial conditions that are replaced with endogenous estimates in later years and the year 2000 measured arguments that remain unchanged or are altered in future years to accord with a revised growth management policy. We have already covered the development of the housing price indices based on the year 2000 observed values. However, we have yet to explain the **Rzone neighborhood score** and the **Rzone child friendly neighborhood score**.

The **Rzone neighborhood score** provides a replicable measure of neighborhood quality. Based on sales of single family homes we construct it as follows:

Rzone neighborhood score = Estimated House Price – Estimated Value of Nonland Components (ie house size, access, age, etc.).

In the above procedure we estimate the component of house value that owes to the land as opposed to the house structural attributes and access. We then normalize this value to an index varying between 0 and 1.

Comparison of developmental model HIAK allocations with actual estimated HIAK data indicated a good match with the exception of number of households with school age children present. Examination of other purchasing and voting data correlated with census tracts with high and low numbers of children unrelated to arguments already in the housing preference and location functions indicated that there was another dimension to the housing preference function. Moreover, this dimension was only relevant to households with school age children present. We defined it as:

Rzone child friendly neighborhood score = (Rzone percent households in year 2000 with school age children present/ Average Rzone percent households in year 2000 with school age children present)

As implemented in the model this variable operates as an addition to neighborhood score only for households that have school age children present.

At present **Rzone neighborhood score** and **Rzone child friendly neighborhood score** are used as fixed, exogenous variables that are held constant throughout a MetroScope run. We anticipate they may be changed up or down to better match an anticipated policy change. However, the amount of the change should be carefully considered and made commensurate with the extent and feasibility of the proposed policy.

MetroScope Residential Behavioral Procedures and Equations:

Below we list the behavioral (non identity) procedures and equations used in MetroScope. The good news is that structurally we have relatively few equations but they are used many times with different parameters for various housing types. In the equation listing below we display the equations for OSFD only since structurally the equations (not parameters or variable values) are the same for the other housing types. Rather than differ between accounting identities and behavioral equations we continue the same equation accounting convention.

HIAK Rank Order Housing Consumption Procedure – Order HIAK classes by housing consumption level at regional geography. The following procedure is necessary to rank order the 400 HIAK classes by their expected average housing consumption. For our purposes this is done once for each 5 year period and not changed. Later in the model the rank order allows HIAK classes to be assigned to 8 equal classes of housing consumption by tenure choice and Ezone work location. Here we characterize housing consumption in terms of house size and lot size. Later in the model all these equations are repeated with tenure, house type and Rzone specific information. In this initial instance however, we use regional averages with a price index set to one to obtain the regional rank order for HIAK classes. House price indices here are shown as exogenous, because we are using measured year 2000 data as arguments in these equations. In all subsequent years price indices are endogenous.

R 12. Determine **year 2000 regional average lot size by HIAK class**
= [household size, income, age, k-12 child?]

R 13. Determine year 2000 regional average house size by HIAK class
= [household size, income, age, k-12 child?]

R 14. Determine year 2000 regional average house price (OSFD) by HIAK class.
= [lot size, house size, yr 2000 % attached, yr 2000 average neighborhood score, yr 2000 average access index]

In equation 14 we do not use a price index since the calibration year is 2000 and the price index is embedded in the constant of the equation.

Rank Order Sort – Determine rank order of HIAK classes by sorting in ascending order the average they are willing to pay for SFD housing in year 2000. This remains constant throughout the model for each 5 year forecast period.

Residential Model – Demand by Tenure, House Type and Rzone Location Procedure – This module determines demand numbers by HIAK class for first tenure, then house type and finally location by Rzone. The allocation process is ordered by place of employment Ezone so all variables are expressed from the measurement perspective of working in a particular Ezone.

R 15. Determine number of households in each HIAK class supported from each employment zone.
= [Employment in zone] divided by [Total employment] times [Regional total in each HIAK class]

Note here that this innocent equation builds in the assumption that each Ezone supports an identical HIAK distribution. In reality wage rates and employment type/occupation class do vary between Ezones. However, departing from this assumption involves wage rate determination equations and wage equilibration in both the residential and nonresidential modules as well as IPF routines that conserve regional HIAK control totals. Departing from the constant HIAK distribution assumption would add at least an order of magnitude of complication to both the residential and nonresidential modules within MetroScope. Looking at wage rate variation at the County level we saw some but not very much¹⁷. As a consequence, at the time we felt these improvements to the model required far more work than the return in accuracy merited.

R 16. Determine tenure choice of household in each HIAK class supported from each employment zone.
= [Households in each HIAK class by Ezone] times {[household size, income, age, k-12 child?, price of substitute, price of choice, price of complement, price of anti-complement, frequency of choice]}

¹⁷ This is after correcting for employment sector. 90% or more of wage rate variation owes to employment type; not regional locational effects.

Housing Consumption Bin Assignment Procedure – This module assigns owners and renters by HIAK class and Ezone to one of eight “consumption bins”. Divide total owners and renter assigned to each Ezone by 8. This gives the number of owners and number of renters in employed in each Ezone that should be in each consumption bin. Assign HIAK classes to consumption bins using existing rank order. For instance, if owners in Ezone 101 require 5000 households per class, the first 5000 households of owners employed in Ezone 101 would go to bin 1; the next 5000 would go to bin 2...etc. Here the consumption bin assignment is recalculated with every iteration of the residential model but the rank order of HIAK’s does not change.

R 17. Determine house type choice of household in each HIAK class and tenure supported from each employment zone.

$$= \text{[Households in each HIAK class by Ezone by tenure]} \text{ times } \{[\text{household size, income, age, k-12 child?, price of substitute, price of choice, price of complement, price of anti-complement, frequency of choice}]\}$$

Note: At this point the residential demand model has determined for each Ezone the number of households supported from it; their choice of tenure and their choice of housing type. The HIAK’s for each employment zone have been rank ordered by housing consumption and assigned to one of eight consumption bins. Next the residential demand model proceeds to allocate each Ezone’s households to a particular residential location.

R 18. Determine location choice (Rzone) by tenure, by house type, by Ezone for each HIAK class by consumption bin.

$$= \text{[Household in each HIAK class by Ezone by tenure by house type]} \text{ times } \{[\text{price of choice, price of complement, price of anti-complement, travel time, neighborhood score, child friendly neighborhood score, household size, income, age, k-12 child?, Rzone bin frequency}]\} \text{ divided by } \{ \text{sum over all rzones by Ezone by HIAK class} [\text{price of choice, price of complement, price of anti-complement, travel time, neighborhood score, child friendly neighborhood score, household size, income, age, k-12 child?, Rzone bin frequency}] \}$$

Note: In the above equation we ultimately compare for a household supported from a particular Ezone the attractiveness of a particular Rzone relative to all Rzones. If we omit the Rzone bin frequency variable, we measure the utility of one dwelling unit in the Rzone relative to all other Rzones. When we include the Rzone bin frequency weighting we directly account for the number of units of that consumption level (the relevant choice set) that exist in the Rzone. Consequently, a Rzone with twice as much utility but ½ as many units in a particular consumption bin has the same attractiveness as a Rzone with ½ the utility but twice as many units. Also keep in mind the Rzone bin frequency is given by bin. Consequently, the attractiveness of a Rzone for a HIAK class in bin 8 is only influenced by the number of dwelling units that are in bin 8 in the Rzone.

R 19. Determine house price by tenure, by type, by Rzone, by HIAK class.

$$= \text{[lot size, house size, Rzone supply price by house type for period t, Rzone access index, Rzone neighborhood score]}$$

Note: The above equation is structurally a repeat of the equation used to order the HIAK's by housing consumption at the regional level. Here the equation works at the Rzone level by housing type with most of the arguments endogenous. Also note that at the Rzone level the allocations of households by size, income, age and k-12 child present are endogenous though the total of the households at the regional level remains a fixed exogenous input from the regional econometric model. Lot size and house size here are the fixed amounts for the region by HIAK class. In other words consumption bin 1 for OSFD presumes a 750 sq. ft. house on a 2500 sq. ft. lot for all housing throughout the region though the price of the house may vary over a factor 2 – 4 depending on location.

Surprisingly enough, we have but eight structural behavioral equations on the residential demand side. Most of the model complexity involves running the same stuff for a number of different classes with different variable and parameter values.

Residential Supply Module:

First it is most important to understand the theoretical economic viewpoint of the supply module. We conceptualize the supply side of the residential real estate market as consisting of large numbers of competitive, independent firms behaving as myopic price takers. More specifically, each supplier knows the output capacity of the land, the cost of production and the price the market is willing to pay for a given housing product. No information is assumed or given about the overall extent of housing demand or future market conditions. Given this, suppliers continue to build housing of any type and configuration in any given location as long as the demand price by housing type, consumption bin and location exceeds the cost of production (plus nominal profit) by housing type, consumption bin and location with the proviso that government regulations permit the housing type.

In terms of communication between the residential demand module and the supply module the exchange is limited to the following. In each model iteration the demand module passes to the supply side the demand price by housing type, consumption bin and location. The supply module in turn passes to the demand module the number of housing units constructed by housing type, consumption bin and location. Vacancy rate by housing type and location is give as an accounting identity of supply less demand. Consistent with economic theory the model operates to minimize the vacancy rate consistent with the behavioral and policy information available to the model.

Calculate Real Estate Price Change Relative to Calibration Price, Land Price, Land Quantity Used and Revised Capital/Land Ratios

The first section of the supply module reads the revised supply price at each iteration and uses that information to update the change in land used, lot prices and the amount of land zoned for the housing type that is actually available to the market. What we term the supply price amounts to the ratio of the current location price to the calibration year location price for the Rzone and housing type. These changes are applicable to all land by housing type within an Rzone.

However, lot sizes are allowed only to change within a specified range in accordance with zoning provisions.

- R 20. Land Price Change as Fraction of Calibration Land Price per sq. ft. by Rzone and House Type = (Log linear function of Rzone Supply Price by House Type)
- R 21. Total capital \$ to total land \$ ratio by Rzone by house type = (kfactor, base capital/land ratio, capital-land substitution parameter)
- R 22. Revised Lot Size by Rzone by house type = (base Rzone lot size by house type/Total capital \$ to total Land \$ ratio by house type by Rzone).
- R 23. Percent Change by Rzone by house type in Lot Size (relative to calibration base) = (Revised lot size/calibration base lot size).
- R 24. Percent change lot price = [(Percent change in land price per sq. ft.) times (1+percent change in lot size)]-1
- R 25. Sales likelihood percentage of all available land in Rzone zoned by house type = (base year percentage of land available) times (land price change raised to an exponent value of 1.5)

The adjustments above are generated by a change in the Rzone real estate price by house type in a given iteration. Changes are relative to an observed calibration price and the changes are applied to all zoning classes and consumption bins within the Rzone for a given housing type. Data that are estimated or assumed for the base year calibration are shown as exogenous.

Demand Side and Initial Condition Supply Side Data Available to Supply Module

This section communicates the demand data available to the supply module and reads in the base year initial condition data plus the updated accounts from the land supply accounting section of the supply module. For any given iteration in a particular time period we have:

- R 26. Average house size by Rzone by house type by consumption bin = (Average from demand side by Rzone by house type for that consumption bin)
- R 27. Average house price by Rzone by house type by consumption bin = (Average absolute price from demand side by Rzone by house type by consumption bin)
- R 28. Total acres potentially available at time period t by Rzone by housing type and by zone class = (Total acres remaining from prior time period by Rzone by housing type and by zone class)

- R 29.** Total acres actually available to the market at time period t by Rzone by housing type and by zone class = (Total acres potentially available...) times (sales likelihood percentage by Rzone by house type)
- R 30.** Average lot size by Rzone by house type by zone class¹⁸ = (mid point of zone class range at calibration price) times (1 + percent change in lot size) **subject to not exceeding lot size range established for zoning class.**
- R 31.** Capacity in total dwelling units by Rzone by house type by zone class = [(Total acres actually available to the market at time t...) times (gross buildable acres to net acres by Rzone by zone class assumption)]/(average lot size by Rzone by house type by zone class)
- R 32.** Land cost per lot by Rzone by house type by zone class = (base price at calibration by Rzone by house type by zone class) times (1 + percent change in lot price)
- R 33.** Net SDC/building fee (subsidy) per lot by Rzone by house type by zone class = (initial condition baseline policy assumption)

In any given iteration of the supply module the above endogenous data are updated. Other initial condition data such as gross to net ratios and development fees or subsidies may be changed prior to each model run. Similarly additional land during the run may be added to the model by modifying the input files prior to the beginning of a model run. For instance additional UGB expansion or redevelopment and infill land may be recognized in a particular forecast year.

Procedure for Calculation of Supply Feasibility by Consumption Bin; Determining Supply Utility by Consumption Bin; Allocation of New Supply by Consumption Bin and Land Accounting Identities.

Once we have provided the supply module with the demand price information and the dwelling unit size information, the supply module calculates the production cost by consumption bin, Rzone and zoning category.

- R 35.** Dwelling unit improvement cost by Rzone, by house type, by zone class, by consumption bin = (function -dwelling unit size by consumption bin/base dwelling unit size) times (cost per sq. ft. by Rzone, by zone class, by house type) times (dwelling unit size by Rzone, by house type, by zone class, by consumption bin)
- R 36.** Total dwelling unit cost by Rzone, by house type, by zone class, by consumption bin = (lot cost by rzone, by house type, by zone class) + (development fee (subsidy)) + (dwelling unit improvement cost by Rzone, by house type, by zone class, by consumption bin).

¹⁸ Average lot size by consumption bin is also calculated on the demand side but that value is the average for a consumption bin that may extend over several zone classes. The lot size here is the lot size within a specified zone class size range. The size of the house however, is taken directly from the consumption bin average since that value remains constant with respect to zone class size and only changes by Rzone and consumption bin.

R 37. Total dwelling units feasible to build by Rzone, by house type, by zone class by consumption bin = (demand price by Rzone, by house type, by consumption bin greater than total dwelling unit cost), (If yes then total capacity by Rzone, by house type, by zone class).

At this juncture it is helpful to point out that the supply module is behaving in a myopic, price taking fashion. What we mean is that for each feasible consumption bin (and there may be as many as eight feasible consumption bins for one zone class) the model uses up all of the available capacity. In some cases this assumption that we have embedded into the algorithmic procedures results in 8 times more building than capacity allows. Again, this seemingly odd procedure follows directly from our strict requirement that many, competitive suppliers do not know the extent or character of demand but they do know the demand price, supply capacity and cost of land and construction in any given iteration. Obviously, despite our theoretical clarity we cannot exceed the total available capacity in any zone class within any Rzone. Consequently, we need prorate excess building back to the level where supply of the zone type capacity is just used up. To do this we use the following procedure.

R 38. Supply utility by Rzone, by house type, by zone class, by consumption bin = (function – demand price/supply cost) times (total dwelling units feasible by zone class by consumption bin)

R 39. Actual supply allocation by Rzone, by house type, by zone class, by consumption bin = (supply utility of consumption bin by zone class/total supply utility of all consumption bins by zone class)

This latter procedure recognizes that consumption bin classes offering higher profit margins will induce suppliers to be more aggressive and take a greater than proportional share of the available capacity for any given zone class. Also, it is not always the case, particularly for RMFD dwelling units that the largest profit margins are in the largest consumption bins. Though as intuition would suggest, larger consumption bins do usually have higher profit margins particularly for OSFD type products.

At this juncture for any given iteration of the residential module, we have determined the amount of dwelling units supplied by Rzone, house type, zone class and consumption bin. Next we need to calculate the amount of land that was used and the amount of land that remains available at the end of each iteration of the residential module.

R 40. Acres used by Rzone by house type by zone class = (potential capacity in DU/actual supply allocation in DU) times (acres actually available to market)

R 41. Acres remaining by Rzone by house type by zone class = (Total acres potentially available) minus (Acres actually used)

Conclusion – Residential Demand and Supply Modules – Equilibrium Iteration

Up to this point we have stepped through a narrative, general description of the residential demand and supply equations. Vital to the equation system is that each equation can be solved in terms of one unknown: price. Keep in mind here that we are not talking about parameter estimation – parameters for each of the equation arguments have already been estimated and now the system constitutes a set of simultaneous equations that purport to measure the state of urban residential real estate once we have an equilibrium solution for price. Specifically, we find:

R 42. Location price by Rzone and house type in forecast period $n = (\text{Location price previous iteration}) \text{ times (adjustment factor)}$.

Min: $(\text{Supply} - \text{Demand} - \text{sqr.d.})$

Subject to: $(\text{Supply by Rzone by house type})$ greater than $(\text{Demand by Rzone by house type})$

The algorithm we employ to carry out this computation is quite simple. For any given Rzone and house type we determine if supply exceeds demand or visa versa. If we have too much supply the location price is decreased by an adjustment factor that is some function of the difference between supply and demand. This has the effect of decreasing supply and increasing demand. If demand exceeds supply, the location price is increased by an adjustment factor that is some function of the difference. This decreases demand and increases supply. For each iteration of the residential model, location prices are changed for each Rzone and housing type. Acceptable convergence of supply and demand occurs within 25 – 50 iterations.

Experience has shown us that it is far more efficient to iterate a fixed number of iterations rather than iterate below a fixed sums of squares criterion. In this fashion replicating and duplicating a run is straightforward and the time of the run is known beforehand. At the completion of a fixed set of iterations the residential model passes the requisite household location information to the nonresidential model.

Nonresidential Model – Equations in English (Almost) Demand Module:

As pointed out at the start of the detailed equation section, the nonresidential model uses 72 Ezones that are coterminous combinations of census tracts. Like the residential model there is no restriction on the number of Ezones within MetroScope. The upper limit on number of zones is established by computational time, volume and complexity of output and the tolerance of the model for data error. Compared to the residential model employment data are far more heterogeneous and subject to various data errors in geocoding. Large zones minimize these errors. Also, since nonresidential zoning and locations are relatively limited in extent, the presence of nonresidential real estate is much more discontinuous as zone sizes decrease. These

discontinuities pose calibration challenges to the model as otherwise identical zones are totally different in regard to employment location.

As in the residential model the number of simultaneous equations that must be solved at each iteration is determined as the number of zones times the number of real estate types. For the nonresidential model we have 72 Ezones times 6 real estate types yielding 432 equations that are solved for location prices in each 5 year period.

Demand by Ezone, Employment Class and Real Estate Type

Like the residential model the nonresidential model takes a 6 county estimate of employment in 15 classes and allocates it to Ezones and 6 real estate types within Ezones. The generalized arguments in verbal terms are as follows.

- E 1. Raw sq. ft. required by Ezone by employment type by real estate type = (region wide total of employment type) times (baseline percentage allocation of the employment type to the real estate type) times (price of real estate type by Ezone, prices of competing real estate types by Ezone) times (base sq ft. per employee by emp type and real estate type, price of real estate type) times (access to hh, access to total employment, access to same employment type, real estate price of zone)
- E 2. Raw employment by Ezone by employment type by real estate type = (raw sq. ft. by Ezone, by emp type....) / (base sq ft. per employee by emp type and real estate type, price of real estate type)
- E 3. Adjustment factor by employment type = (regional control total for employment type)/(sum of raw employment over Ezone and real estate type)
- E 4. Adjusted sq. ft. required by Ezone by employment type by real estate type = (raw sq. ft. required by Ezone...by real estate type) times (adjustment factor by employment type)
- E 5. Adjusted employment by Ezone by employment type by real estate type = (raw employment required by Ezone...by real estate type) times (adjustment factor by employment type)

Note that the above equations require an initial condition estimate of how a given employment class at the regional level is distributed among the 6 nonresidential real estate types. Likewise, the equations require at the regional level an initial condition of the square feet per employee requirements by employment class and real estate type. As presently configured these initial data requirements amount to 2 15 by 6 matrices of percentages and square feet per employee estimates that we provide using sample survey data.

Besides price of the real estate, nonresidential demand is driven by three access indices calculated for each Ezone relative to all other Ezones within the economic region. The three access indices are defined as follows:

- E 6. **Access to households** = sum over all Ezones: (number of households in Ezone/travel time to Ezone, parameters)
- E 7. **Access to total employment** = sum over all Ezones: (total employees in Ezone/travel time to Ezone, parameters)
- E 8. **Access to the same employment type** = sum over all Ezones: (total same emp class employees in Ezone/travel time to Ezone, parameters)

The functional form of the nonresidential demand equations is presently pseudo-translog. We include direct price and cross price elasticities between all real estate types as well as output effects measured in terms of total employment demand by class. Real estate choice cross price elasticities are constrained to be symmetrical and sum to zero. Direct price elasticities are negative and cross elasticities are positive. Demand with respect to prices (not access) is specified to be homogenous but not necessarily of degree one.

Coefficients associated with the 3 access index terms are constrained to be positive and sum to 1.

The direct price elasticity for location choice by employment type is constrained to a value range of -.2 through -2.

The above set of equations constitute the demand side of the nonresidential model. Operating as such the model produces estimates of demand by square footage and employment by Ezone, employment type and real estate type.

Supply by Ezone and Real Estate Type

The supply module for the nonresidential model works much like the supply module in the residential model with one important exception. In the residential model there is a supply side for every housing type. This is not so in the nonresidential model. Here the 6 real estate types on the demand side are supplied from 3 land sources. Manufacturing and warehousing real estate types draw from the “industrial” land base. Retail and services are supplied from the “commercial” land base. And medical/social and government are supplied from the “institutional” land base.

Another convention that we use in the nonresidential model (also used in the residential model) is the restriction in real estate usage to the zoning of the type of land you are located on. What we mean by this is that manufacturing or warehouse type real estate must be built on “industrial” land. Similarly, retail/services and general office are only built on “commercial” land. Like the residential model, the nonresidential model assumes land use zoning is totally exogenous. However, as is clear in the model output any employment type (ie construction, agriculture-mining, business services, K-12 education, etc.) may locate in any of the 6 real estate categories. This allows warehouse space to be occupied by say a dance studio or any insurance claims adjuster (just 2 of many enterprising uses I have noticed of unused industrial space). As was explicit in the demand equations, use of space by any employment type directly responds to the price for the real estate class being considered and the prices of all competing classes.

Consequently, in areas such as the Central Eastside where commercial and retail demand is high and manufacturing/warehousing demand is relatively low, non industrial uses move into industrial type space.

Unlike the residential model, the nonresidential model generalizes jurisdiction nonresidential zoning into 8 FAR classes. These are 0 - .25, .25 - .5, .5 - .75, .75 - 1, 1 - 1.5, 1.5 - 2.0, 2.0 - 5.0, 5.0 +. As noted above “industrial”, “commercial” and “institutional” comprise the exclusive land classes.

Non Residential Supply Equations in Narrative Form

- E 9.** Total acres by land class by FAR by Ezone = (vacant acres by land class by FAR by Ezone) + (refill acres by land class by FAR by Ezone)
- E 10.** Acres available in forecast year by land class by FAR by Ezone = (total acres by land class by FAR by Ezone) times (time period market factor)
- E 11.** Time period market factor = (assumed constant percentage) or (function: (time period price/calibration price)) > 0, <1.
- E 12.** Baseline construction cost per sq. ft. by land class by FAR by Ezone = (land cost by FAR by land class by Ezone per sq. ft.) + (capital cost by FAR by land class by Ezone per sq. ft.)
- E 13.** Potential sq. ft. of capacity by land class by FAR by Ezone = (mid point of base line FAR by land class by Ezone) times (acres available in forecast year by land class by FAR by Ezone) times (sq. ft. in acre).
- E 14.** Base price per sq.ft. by real estate type by FAR by Ezone = (annual rent per sq. ft. – annualized over 20 years)
- E 15.** Price ratio by real estate type by Ezone = (demand side – current location price/calibration location price by real estate type by Ezone)
- E 16.** Current (new) price per sq. ft. by real estate type by FAR by Ezone = (base price per sq. ft.) times (price ratio by real estate type by Ezone)
- E 17.** Current (new) land price ratio = (higher of 2 current price ratios, function – exp(b*price ratio) with b>1.0.
- E 18.** Current (new) FAR by land use class by FAR by Ezone = (function - capital/land ratio cost per sq. ft. , new land price ratio, capital land substitution parameter)
- E 19.** Current (new) capacity in sq. ft. by land class, by FAR, by Ezone = (current (new) FAR by land use class by FAR by Ezone) times (acres available by land use class by FAR by Ezone).

- E 20. Current (new) construction cost per sq. ft. by land use class by FAR by Ezone = (capital cost per sq. ft. by land use class by FAR by Ezone) + [(base line land cost per sq. ft. ...by Ezone) times (new land price ratio)].
- E 21. Initial demand in sq. ft. by real estate type by FAR by Ezone = (Demand side current iteration data, Current price by real estate type> current construction cost)
- E 22. Initial prorate of real estate types of current capacity by land class = (current (new) capacity in sq. ft. by land class, by FAR by Ezone) times (demand in sq. ft. by real estate type/sum of demand by the 2 real estate types using the land class)
- E 23. Final assignment of sq. ft. by real estate type by FAR by Ezone = (procedure – check to make sure initial prorate does not exceed available capacity).

Iteration of Supply and Demand in the Nonresidential Model

The nonresidential model achieves a balance of demand and supply using the same basic approach as the residential model. Specifically, we seek a “location price” by Ezone and

real estate type that minimizes the squared difference between supply and demand for each of the 6 nonresidential real estate types in each Ezone.

- E 24. Location price by Ezone and real estate type: That

Min: (Supply – Demand – sqrd.)

Subject to: (Supply by Ezone by real estate type) greater than (Demand by Ezone by real estate type)

Model Equations

Residential Module - Weighted Price and Frequency Accounting Identities:

Accounting Identities – Rzone Based

R 1.- R 4.)

$$Loc\ Price_{Rzone,Year}^{HouseType} = [PriceIndex_{Rzone,Year2000}^{HouseType}] \times [Loc\ Price_{Rzone,Year}^{HouseType} Adj.factor]$$

R 5.)

$$Supply\ Price_{Rzone,Year}^{HouseType} = [Loc\ Price_{Rzone,Year}^{HouseType}] \sqrt{[Loc\ Price_{Rzone,Year2000}^{HouseType}]}$$

These equations are computed for all four housing types over all Rzones for every forecast period. The location adjustment factor is provided by the supply-demand minimization algorithm as the real estate module is solved iteratively for the vector of location prices that minimizes the squared difference between supply and demand in each Rzone by housing type. Note that the original calibration year price index drops out of the calculation after the calibration year.

The Supply Price equations simply calculate the ratio of change in price for any given forecast year relative to the base year calibration price. Supply prices are used to change the hedonic housing price estimates on the demand side and to provide information to the supply module for determining how much land is in the market and the ratio of capital to land used in housing production.

The weighted tenure location price is calculated from the housing type location price using the equation below.

R 6.)

$$Loc Price_{Rzone,Year}^{Rent} = \frac{\left[(Loc Price_{Rzone,Year}^{HouseType:RSFD}) \times (DUSupply_{Rzone,Year}^{HouseType:RSFD}) + ((Loc Price_{Rzone,Year}^{HouseType:RMFD}) \times (DUSupply_{Rzone,Year}^{HouseType:RMFD})) \right]}{\left[(DUSupply_{Rzone,Year}^{HouseType:RSFD}) + (DUSupply_{Rzone,Year}^{HouseType:RMFD}) \right]}$$

An identical calculation is made to obtain the weighted Rzone location price for the owner tenure.

Accounting Identities – Ezone

Once the Rzone specific location prices are calculated for each housing type and tenure for each time period¹⁹ we use them to compute a set of

¹⁹ Actually these price indices are recalculated for each iteration of the residential model.

price indices that are aggregated and weighted from the perspective of each Ezone.

$$\begin{aligned}
 & \text{R 7.) } \text{Loc Price}_{Ezone,year}^{HouseType} = \sum_{Rzone} [(\text{Loc Price}_{Rzone,year}^{HouseType} \times \text{DU}_{Rzone,year}^{HouseType}) / \\
 & (\text{TravelTime}_{EzoneRzone,year})] / \sum_{Rzone} [\text{DU}_{Rzone,year}^{HouseType}] / \\
 & (\text{TravelTime}_{EzoneRzone,year})]
 \end{aligned}$$

What is special here is that the Ezone specific indices are weighted by the travel time to each Rzone in the region. Consequently the location prices and numbers of dwelling units close to an Ezone contribute the most information. (In fact if it were not for the travel time weighting; the index would be identical for each Ezone.)

This accounting equation is repeated for each housing type for each Ezone. We then use the weighted location price by Ezone to form the weighted price index by tenure for each Ezone by combining the RMFD and RSFD location prices for renter tenure and the OMFd and OSFD location prices for owner tenure.

$$\begin{aligned}
 & \text{R 8.) } \text{Loc Price}_{Ezone,year}^{Tenure} = \sum_{Rzone} [(\text{Loc Price}_{Rzone,year}^{HouseType} \times \text{DU}_{Rzone,year}^{HouseType}) \\
 & + (\text{Loc Price}_{Rzone,year}^{HouseType} \times \text{DU}_{Rzone,year}^{HouseType})] / \sum_{Rzone} [(\text{DU}_{Rzone,year}^{HouseType}) + (\text{DU}_{Rzone,year}^{HouseType})]
 \end{aligned}$$

In the above equation “HouseType” is OSFD and OMFd for owner tenure and RSFD and RMFD for renter tenure. Note that in the above equation we can drop travel time since it is already accounted for in the weighted Ezone location price equation.

Similarly we can form an Ezone total housing location price index by combining the 2 tenure location prices.

R 8a.)

$$Loc Price_{Ezone,year}^{Total} = \sum_{Rzone} [(Loc Price_{Rzone,year}^{own} \times DU_{Rzone,year}^{own}) + (Loc Price_{Rzone,year}^{Rent} \times DU_{Rzone,year}^{Rent})] / \sum_{Rzone} [(DU_{Rzone,year}^{Own}) + (DU_{Rzone,year}^{Rent})]$$

Housing Frequency, Auto and Transit Cost Indices –

From an Ezone perspective we also calculate housing frequency, auto cost and transit cost by housing type and tenure. Since we are presently not using auto and transit cost in these arguments, we do not detail their computation though it is identical in structure to housing frequency.

R 9.)

$$HouseFreq_{Ezone,year}^{Tenure} = \sum_{Ezone} \sum_{Rzone} [DU_{Rzone,year}^{HouseTypeOSFD}] / TravelTime_{EzoneRzone,year}] + \sum_{Ezone} \sum_{Rzone} [DU_{Rzone,year}^{HouseTypeOMFD} / TravelTime_{EzoneRzone,year}]$$

A weighted Ezone house frequency variable is constructed for renter tenure using the same equation as in R 9 but with RSFD and RMFD DU values.

R 10.)

$$HouseFreq_{Ezone,year}^{HouseType} = \sum_{Ezone} \sum_{Rzone} [DU_{Rzone,year}^{HouseType}] / TravelTime_{EzoneRzone,year}]$$

Equation R 10 is for a specific house type. In reality it is calculated before the composite tenure equation in R 9. However, since we choose tenure prior to type the composite frequency appears first in the equations so we have retained that order for the accounting identities.

What is most critical in the Ezone based price and frequency variables is that the aggregate of regional prices and frequencies for various housing types and tenures change by Ezone since they are weighted (inversely) by travel time. Consequently, someone working in the CBD sees a considerably different tenure choice landscape, than does someone working outside the UGB.

Rzone Bin Frequency (Odds Ratio)

The equation below defines the market segmentation of each of the 4 housing types in each Rzone into 8 market segments. What we do is take the total number of units in each of the 8 market segments (determined by 8 values of regional average housing price) and divide that number for each Rzone by the regional average of all Rzones. So an Rzone with twice as many housing units in a market segment (value class) as the regional average for an Rzone would have a value of 2.

$$R\ 11.)\ FreqChoice_{ConBin,Rzone}^{HouseType} = \left[DU_{ConBin,Rzone}^{HouseType} \right] / \left[\frac{1}{n_{Rzone}} \sum_{Rzone}^n DU_{ConBin,Rzone}^{HouseType} \right]$$

This value is used in the location equations to determine the share of households in a given market segment that will be allocated to a particular Rzone in any given iteration. We should emphasize that for reasons of model stability we use the above variable on a lagged basis. (IE its value does not change during the iterative procedure for finding a model solution.)

Demand Side Equations

HIAK Rank Ordering of Housing Consumption at Regional Level

The first equation procedure involves

$$\text{R 12.) } \ln(\text{Regional Average SFD Lot Size by HIAK}) = \\ \ln(A_o) + b_1 \ln(\text{HHsize}) + b_2 \ln(\text{Age}) + b_3 \ln(\text{Age})^2 + b_4 \ln(\text{Income}) \\ + b_5(\text{Kids}) + b_6 \ln(\text{Income})(\text{Kids})$$

$$\text{R 13.) } \ln(\text{Regional Average SFD House Size by HIAK}) = \\ \ln(A_o) + b_1 \ln(\text{HHsize}) + b_2 \ln(\text{Age}) + b_3 \ln(\text{Age})^2 + b_4 \ln(\text{Income}) \\ + b_5(\text{Kids}) + b_6 \ln(\text{Income})(\text{Kids})$$

$$\text{R 14.) } \ln(\text{Regional Average SFD House Price by HIAK}) = \\ \ln(A_o) + b_1 \ln(\text{Lotsize}) + b_2 \ln(\text{Hsesize}) + b_3 \ln(\text{Ave RegAccessIdx}) \\ + b_4(\text{AttachedHse?}) + b_5(\text{Ave RegNeighborScore})$$

Procedure:

Rank order of HIAK ascending by Regional Average SFD

House Price=Sort:Ascending(SFD House Price)

A unique aspect of the residential real estate module of MetroScope is the division of the market among 4 housing/tenure types and then the further division of each housing/tenure types into 8 value classes. Both supply (the housing stock) and demand (the 400 HIAK classes) are segmented into 8 value classes. This approach avoids the large amount of noise contributed by irrelevant choices. Consequently, in MetroScope choice of location is only influenced by the computed utility of the location and the number of housing choices that are in a particular value classes' market segment.

The rank order of HIAK classes is a constant that is applied during the model run to the number of owners and renters in a particular iteration. The array of owners or renters is divided into 8 equal classes and the resultant classes represent the 8 value classes that are then matched to the supply side value distributions in each Rzone.

Determine Residential Demand by Tenure, House Type and Rzone Location

R 15.) Number of HH by HIAK “working” in each Ezone =

$$\left[\left(\frac{Emp_{Ezone}}{TotalEmp} \right) \times (TotalHH_{HIAK}) \right]$$

R 16.) Utility of Own Tenure by HIAK by Ezone =

$$\exp \left[\begin{aligned} & (Ln A_o + b_1 Ln(Age_{HIAK}) + b_2 Ln(Age_{HIAK}^2) + \\ & b_3 Ln(Income_{HIAK}) + b_4 Ln(Income_{HIAK}^2) + \\ & b_5 Ln(HHsize_{HIAK}) + b_6 (Kids_{HIAK}) + \\ & b_7 Ln(Loc Price_{Ezone}^{Rent}) + b_8 Ln(Loc Price_{Ezone}^{Own}) + \\ & b_9 Ln(PriceTransComplement_{Ezone}) + b_{10} \\ & Ln(PriceTransSubstitute_{Ezone}) + \\ & b_{11} Ln(HouseFreq_{Ezone}^{Tenure}) \end{aligned} \right]$$

Please note that the independent arguments in these equations are calculated from the perspective of each Ezone (place of employment) rather than Rzone. The calculation of the price variables we covered in the previous section. The equation is a binomial equation for own or rent choice.

$$R 16a.) P_{HIAK,Ezone}^{Own} = U_{HIAK,Ezone}^{Own} / (1 + U_{HIAK,Ezone}^{Own})$$

$$R 16b.) P_{HIAK,Ezone}^{Rent} = 1 - P_{HIAK,Ezone}^{Own}$$

$$R 16c.) HH_{HIAK,Ezone}^{Own} = [HH_{HIAK,Ezone}^{Total}] \times [P_{HIAK,Ezone}^{Own}]$$

$$R 16d.) HH_{HIAK,Ezone}^{Rent} = [HH_{HIAK,Ezone}^{Total}] \times [P_{HIAK,Ezone}^{Rent}]$$

R 17.) Utility of SFD or MFD by HIAK by Ezone =

$$\exp \left[\begin{aligned} & (\text{Ln } A_o + b_1 \text{Ln}(\text{Age}_{\text{HIAK}}) + b_2 \text{Ln}(\text{Age}_{\text{HIAK}}^2) + \\ & b_3 \text{Ln}(\text{HHsize}_{\text{HIAK}}) + b_4 \text{Ln}(\text{Income}_{\text{HIAK}}) + \\ & b_5 \text{Ln}(\text{KidsxIncome}_{\text{HIAK}}) \\ & + b_6 (\text{PriceTransComplement}_{\text{Ezone}}) + \\ & b_7 \text{Ln}(\text{PriceTransSubstitute}_{\text{Ezone}}) \\ & + b_8 \text{Ln}(\text{PriceChoice}_{\text{Ezone}}) \\ & + b_9 \text{Ln}(\text{PriceSubstitute}_{\text{Ezone}}) + \\ & b_{10} \text{Ln}(\text{HouseFreq}_{\text{Ezone}}^{\text{Type}}) \end{aligned} \right]$$

Please note this is presently a binomial equation modified from an earlier multinomial equation that divides the own choice group into single family detached and multi-family and is repeated again for the renter group with different parameters and argument values. As in the housing tenure choice equation arguments are calculated from the perspective of each of the Ezones.

$$\text{R 17a.) } P_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own}} = U_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own}} / U_{\text{HIAK}}^{\text{MFD,Own}} + U_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own}}$$

$$\text{R 17b.) } P_{\text{HIAK},\text{Ezone}}^{\text{MFD,Own}} = U_{\text{HIAK},\text{Ezone}}^{\text{MFD,Own}} / U_{\text{HIAK}}^{\text{MFD,Own}} + U_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own}}$$

$$\text{R 17c.) } HH_{\text{HIAK},\text{Ezone}}^{\text{Own,SFD}} = [HH_{\text{HIAK},\text{Ezone}}^{\text{Own}}] x [P_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own}}]$$

$$\text{R 17d.) } HH_{\text{HIAK},\text{Ezone}}^{\text{Own,MFD}} = [HH_{\text{HIAK},\text{Ezone}}^{\text{Own}}] x [P_{\text{HIAK},\text{Ezone}}^{\text{MFD,Own}}]$$

$$\text{R 17e.) } P_{\text{HIAK},\text{Ezone}}^{\text{SFD,Rent}} = U_{\text{HIAK},\text{Ezone}}^{\text{SFD,Rent}} / U_{\text{HIAK}}^{\text{MFD,Rent}} + U_{\text{HIAK},\text{Ezone}}^{\text{SFD,Own,Rent}}$$

$$\text{R 17f.) } P_{HIAK,Ezone}^{MFD,Rent} = U_{HIAK,Ezone}^{MFD,Rent} / U_{HIAK}^{MFD,Rent} + U_{HIAK,Ezone}^{SFD,Rent}$$

$$\text{R 17g.) } HH_{HIAK,Ezone}^{Rent,SFD} = [HH_{HIAK,Ezone}^{Rent}]x[P_{HIAK,Ezone}^{SFD,Rent}]$$

$$\text{R 17h.) } HH_{HIAK,Ezone}^{Rent,MFD} = [HH_{HIAK,Ezone}^{Rent}]x[P_{HIAK,Ezone}^{MFD,Rent}]$$

Again, note that we have a hierarchical system of choice. First a tenure choice is made; subsequently a housing type choice. Parameter estimates are constrained such that the probability of making a switch between housing types given a change in the arguments is much greater than a change of tenure. This hierarchy holds true also for the third level – housing location. In other words – the chance of a location change is always much higher than the chance of a housing type change and the chance of a housing type change is greater than a tenure change.

For the first two levels of the hierarchy the arguments for each equation are calculated and summed over the region from the perspective of each Ezone. In the final level of the hierarchy – location choice – we introduce a number of arguments unique to particular Rzones but viewed from the perspective of the Ezone of which commuter travel time is the most salient.

At this point in the model for each of our Ezones we have determined demand for each HIAK class²⁰ by OSFD, OMFd, RSFD and RMFD. We now assign each of these housing types by HIAK class and Ezone to a Rzone. For readability purposes we suppress the housing type superscripts, HIAK subscripts and state the equation only once instead of 4 times.

²⁰ Keep in mind that in each iteration every HIAK class also maps directly to one of eight demand side “consumption bins”.

$$\begin{aligned}
& HH_{HIAK,Ezone,Rzone}^{HouseType} = HH_{HIAK,Ezone}^{HouseType} x \\
\text{R18.) } & \left[U_{HIAK,Ezone,Rzone}^{HouseType} \right] x \left[FreqChoice_{ConBin,Rzone}^{HouseType} \right] / \\
& \left[\sum_{Rzone}^n \left[U_{HIAK,Ezone,Rzone}^{HouseType} \right] x \left[FreqChoice_{ConBin,Rzone}^{HouseType} \right] \right]
\end{aligned}$$

$$\text{R 18a.) } U_{HIAK,Ezone,Rzone}^{HouseType} =$$

$$\exp \left[\begin{aligned}
& (\text{Ln } A_o + b_1 \text{Ln}(\text{Time}_{Ezone,Rzone}) + b_2 \text{Ln}(\text{Time}_{Ezone,Rzone}^2) + \\
& b_3 \text{Ln}(\text{EarnersxTime}_{Ezone,Rzone}) + b_4 \text{Ln}(\text{KidsxTime}_{Ezone,Rzone}) + \\
& b_5 \text{Ln}(\text{AgexTime}_{Ezone,Rzone}) + b_6 (\text{HHsizexTime}_{Ezone,Rzone}) \\
& b_7 \text{Ln}(\text{IncomexTime}_{Ezone,Rzone}) + b_8 \text{Ln}(\text{IncomexNeighscore}_{Rzone}) \\
& + b_9 \text{Ln}(\text{PriceChoice}_{Rzone}) + b_{10} \text{Ln}(\text{PriceTransComp}_{Rzone}) + \\
& b_{10} \text{Ln}(\text{PriceTransSub}_{Rzone}) + b_{11} (\text{KidsxKidsIdx}_{Rzone})
\end{aligned} \right]$$

Note that in the above equations the FreqChoice variable is defined over the 8 supply side consumption bins that correspond to the 8 demand side consumption bins determined by a rank ordering of the 400 HIAK classes. Consequently each Rzone has 8 “odds ratios”. The “odds ratio” that a particular HIAK class uses depends on its rank ordering classification.

Total demand by house type and Rzone is obtained by summing over all HIAK and Ezones.

R 18b.)

$$HH_{Rzone}^{HouseType} = \sum_{Ezone} \sum_{HIAK} HH_{Rzone}^{HouseType}$$

Equation R 18b provides a demand estimate consistent with the supply side housing production information.

Next, we have a procedure to determine at any given iteration what the demand side housing price is.

$$R\ 19.)\ House\ Price_{HIAK, Rzone}^{HouseType} =$$

$$\exp \left[\begin{aligned} &Ln(A_o + b_1 Ln(Lotsize_{HIAK}) + b_2 Ln(Hsize_{HIAK}) + b_3 Ln(AccessIdx_{Rzone}) \\ &+ b_4(NghbrScore_{Rzone}) + b_5(Supply\ Price_{Rzone}^{HouseType}) \end{aligned} \right]$$

At this point we have completed the residential demand side of the model. Important to note is that the only totally exogenous variables that are entered into model so far are the neighborhood score and the “kid friendly index”²¹. All other variables are endogenous, computed in other MetroScope modules or carried over from previous time periods.

Supply Side Equations

R 20.)

$$Land\ Priceratio_{Rzone}^{HouseType} = \exp [A_o + b_1 \ln(Supply\ Price_{Rzone}^{HouseType})]$$

Equation R 20 is linearization of a more complicated expression derived from an underlying CES production function with the coefficient more or less equal to one plus the elasticity of substitution which we presently measure as .6 for single family type dwellings.

$$(Land\ \$ / Capital\ \$)_{Rzone}^{HouseType} = KFactor$$

$$R\ 21.)\ \left[\frac{Base\ \$\ Capitalsq.ft.}{(Base\ \$\ Landsq.ft.xLand\ Priceratio)} \right]^{-\alpha}$$

²¹ We compute this from the 2000 Census as percent of households with k-12 age children present.

$$KFactor_{Rzone}^{HouseType} = \left[\frac{BaseCapitalsq.ft.}{BaseLandsq.ft.} \right] /$$

$$R\ 21a.) \left[\left[\frac{Base\$Capitalsq.ft.}{(Base\$Landsq.ft.xLand\ Priceratio)} \right]^{-\alpha} \right]$$

Note that in the above equation we have suppressed the house type and Rzone scripts to improve readability. The “alpha” superscript is the elasticity of substitution between capital and land. Equation R 21a calculates the new physical ratio of capital to land as a result of the land price change. Equation R 21 then converts the physical change in capital to land ratio to the dollar ratio of land divided by capital. Consequently, a change in the supply price drives a change in both the physical and monetary ratios of land and capital used in housing production.

$$R\ 22.) \left(Lotsize \right)_{Rzone}^{HouseType} = BaseLandsq.ft. / \left[\frac{Land\$}{Capital\$} \right]$$

Equation R 22 provides the new lot size as a result of the change in Supply price.

$$R\ 23.) \left(Landquantityratio \right)_{Rzone}^{HouseType} = (LotSize / BaseLandsq.ft)$$

R 24.)

$$\left(Percentchangelotprice \right)_{Rzone}^{HouseType} = \left[(Landquantityratio)x(Land\ Priceratio) \right] - 1$$

Equations R 21 through R 24 (they may be simplified greatly.) hold the quantity and price of capital constant and calculate the change in price and quantity of land used to produce the capital. In MetroScope capital requirements are generated on the demand side with producers varying the amount of land inputs necessary to produce the desired housing

bundle. From this perspective the producer (supply side) in any given model iteration or forecast period, is faced with supplying fixed housing capital attributes but is able to alter land inputs to achieve maximum profit.

Marketable Land Percentage of All Available Land

MetroScope operates in 5 year cycles over a 30 – 60 year period depending on user preference. Much of the land supply except for UGB adds is potentially available to the model beginning in the first 5 year forecast period. Clearly, the market in any given forecast period does not and is incapable of using all the land potentially available. Infrastructure provision, distance from existing facilities and ownership patterns are some of the controls on when land actually becomes “ripe” for the market. MetroScope does not simulate such detailed transactions as land assembly, infrastructure supply decisions or “perfect foresight” calculations of private land owners. In place of a detailed micro-market simulation MetroScope substitutes an empirical statistical equation that estimates by Rzone and house type how much land is available to the market in any 5 year forecast cycle.

$$R_{25.}) \left(\text{Percentlandinmarket} \right)_{Rzone}^{HouseType} = A_o x(\text{Land Priceratio})^\beta$$

The A_o parameter is set by Rzone and generally reflects experience and judgment about how much land in a given Rzone is actually available for development. In general this value ranges from .05 to .3. The “Beta” parameter is set a 1.5. In essence a 100% increase in land price (about 60% increase in real estate sales price) results in a 180% increase in the base level of land available to the market.

Demand Side and Initial Condition Supply Side Data Available to Supply Module

Here we repeat a number of values computed from the demand side that must be available to the supply side module at each iteration.

R 26.)

$$(AveHouseSize)_{Rzone,consumptionbin}^{HouseType} = \frac{1}{n} \sum Regionalavehousesize_{HIAK}$$

In equation R 26 the “n” is summed over each of the 8 consumption bins that the 400 HIAK classes are divided into. As presently implemented in MetroScope regional average house size by consumption bin and house type is a constant across the region so they equation need not be repeated for each Rzone.

R 27.)

$$AveHousePrice_{Rzone,consumptionbin}^{HouseType} = \frac{1}{n} \sum HousePrice_{HIAK}$$

In equation R 27 house price by HIAK does change by Rzone so the calculation is repeated for each rzone, consumption bin and house type. As in equation R 26 the n refers to the 8 consumption bins; so the procedure provides an average price by consumption bin from the HIAK based data.

Land Accounting, Production Cost and Capacity Calculations

R 28.)

$$TotalAcresAvailable_{Rzone,Zoningclass,t}^{HouseType} = TotalAcresLeft_{t-1} + UGBadds_t + URA_{Rzone,t}$$

Equation 28 expresses a land accounting equation that says the total land resource available in time t in a particular Rzone, house type and zone class is what is left over from the previous period and whatever UGB and Urban Renewal adds have been assumed to occur in the current forecast period that affect that Rzone and zone class. Present model rules only apply the housing subsidy to land inside an Urban Renewal Area. As a consequence, in the model details we track URA land and DU capacity

separately but the calculations are identical otherwise. For that reason we suppress the URA details except at the initial land accounting and the final land accounting for purposes of this description.

$$R\ 29.) \quad \frac{\text{Total Acres in market}_{Rzone, Zoningclass, t}^{HouseType}}{\text{Percent land in market}} = \text{Total Acres Available} \times$$

$$R\ 30.) \quad \text{Avelotsize}_{Rzone, Zoningclass}^{HouseType} = [\text{Zone class midpoint} \times (\text{Land quantity ratio})]$$

S.T.:

$$\text{Avelotsize}_{Rzone, Zoningclass}^{HouseType} \geq \text{regulatory min}$$

$$\text{Avelotsize}_{Rzone, Zoningclass}^{HouseType} \leq \text{regulatory max}$$

$$R\ 31.) \quad \text{DUCap}_{Rzone, Zoningclass, t}^{HouseType} = [(\text{Total Acres in market}_{Rzone, Zoningclass, t}^{HouseType}) \times (\text{Net acres} / \text{Gross acres}_{Rzone, Zoningclass}^{HouseType}) \times 43,560] / \text{Avelotsize}_{Rzone, Zoningclass}^{HouseType}$$

$$R\ 32.) \quad \text{Lot cost}_{Rzone, Zoningclass}^{HouseType} = [\text{Base Lot cost} \times (1 + (\text{Percent change lot price}))]$$

Some of the equations R 29 through R 32 seem redundant with those of the previous section. The difference here is that most of the quantities of the previous section were percent changes or ratios. Here the percent changes and ratios are combined with base data to produce absolute values.

Equation R 29 calculates the amount of land in acres that is actually available to the market by Rzone, house type and zone class. R 30 for each zone class converts the land quantity ratio to an actual estimate of lot size that is consistent with the lot size minimums and maximums established in Metro's 34 classes of generalized zoning. Equation R 31

converts the acreage data into dwelling unit capacity using a gross to net acres conversion factor specific to house type and zone class and the average lot size for that zone class and house type. Equation R 32 determines the actual dollar lot cost by multiplying the observed lot cost in the calibration year by the lot price percentage change variable.

$$R\ 33.)\ SDC_Bldgfee_ (Subsidy)_{Rzone}^{HouseType} = [ExogenousAssumption]$$

Equation R 33 amounts to a reminder that part of the production cost accounting identities are given by assumption. These data are read into the model prior to the start of a run and represent one of the variables that may be altered to carry out policy.

Next we move to the procedure for determining whether production may occur by Rzone, house type and consumption bin. Equation R 34 computes land cost, building fee costs and building construction costs.

R 34.)

$$DUimprovementproduction\ cost_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} = [(AveHousesize)_{Rzone,consumptionbin}^{HouseType} / AveHousesize_smallestbin]^\alpha \times (Construction\ cost\ pers\ qft_{Rzone,Zoningclass}^{HouseType}) \times AveHousesize_{Rzone,consumptionbin}^{HouseType}$$

Note that the first factor in equation R 34 is raised to the power “alpha” which is set to .1 in this case. What this does is modify the cost per sq. ft. to construct a house as a function of its size (here quality is constant with size). In this way larger houses become slightly cheaper to build than small houses. Similarly, the exogenously given cost per sq. ft. may change with zone size and Rzone to reflect observed economies of scale in areas with large tracts of undeveloped land.

R 35.)

$$\begin{aligned}
&DU\text{Totalproduction cost}_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} = \\
&[\text{Lot cost}_{Rzone,Zoningclass}^{HouseType} + \text{SDC_Bldgfee_}(Subsidy)_{Rzone}^{HouseType} + \\
&DU\text{improvementproduction cost}_{Rzone,Zoningclass,consumptionbin,t}^{HouseType}]
\end{aligned}$$

Equation R 35 sums up the total production cost for a particular type of house by Rzone, zone class and consumption bin. In any given iteration of the model demand prices higher than this value will enable production to occur (assuming there is land capacity available). Demand prices lower will prevent production from occurring.

The procedure for determining production feasibility and establishing the initial capacity to be used is as follows:

R 36.)

$$\begin{aligned}
&DU\text{Totalfeasible}_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} = \\
&\text{If : } DU\text{Totalproduction cost}_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} \leq \\
&\text{AveHouse Price}_{Rzone,consumptionbin}^{HouseType} \\
&\text{Then : } DUCap_{Rzone,Zoningclass,t}^{HouseType} \text{ Else : } 0
\end{aligned}$$

Note that at this juncture in the model program, a feasible solution uses up all of the capacity for the entire zone class. Feasible production in all 8 consumption bins would have 8 times the actual available capacity. This odd result follows directly from our assumption of myopic suppliers building all that was feasible to build in any given iteration. From a modeling algorithm perspective we need insert procedures to reconcile this inconsistency.

R 37.)

$$\begin{aligned}
 & DUSupplyUtility_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} = \\
 & [AveHouse Price / DUTotalproductioncost]^\beta \times \\
 & DUCap_{Zoneclass}
 \end{aligned}$$

In Equation R 37 the “beta” value is set at 2.5 to reflect that houses with a larger profit margin will be produced far in excess of those with a smaller profit margin.

R 38.)

$$\begin{aligned}
 & DUActualSupply_{Rzone,Zoningclass,consumptionbin,t}^{HouseType} = \\
 & [DUSupplyUtility_{Zoneclass,consumptionbin} / \sum_n^{consumptionbin} DUSupplyUtility_{Zoneclass}] \times \\
 & DUCap_{Zoneclass}
 \end{aligned}$$

Equation R 38 allocated actual production for each zone class to each consumption bin proportional to the computed utility for each consumption bin in that zone class. Next we sum actual supply by consumption bin so we have a total for each zone class.

This completes the process of determining how many units for each consumption bin for each zone class are produced in any given iteration of the model. Next we need to assess how many acres of land were consumed in the production process and how many total acres are available for consumption in the next forecast period.

R 39.)

$$\begin{aligned}
 & TotalAcresused_{Rzone,Zoningclass}^{HouseType} = \\
 & [DUActualSupply_{Rzone,Zoningclass}^{HouseType} / DuCap_{Rzone,Zoningclass}^{HouseType}] \\
 & \times TotalAcre sin market_{Rzone,Zoningclass}^{HouseType}
 \end{aligned}$$

Note that in equation R 39 all quantities have been summed over the 8 consumption bins.

R 40.)

$$TotalAcresLeft_{Rzone,Zoningclass,t}^{HouseType} = [TotalAcresLeft_{t-1} + UGBadds_t + URAadds_t] - TotalAcresUsed$$

Equation R 40 is an identity that says that the total acres left by house type, Rzone and zoneclass at the end of the period is simply the total acres available at the beginning of the period less that amount that was used during the period. Also keep in mind that in terms of the actual model accounting details on the supply side we track the URA accounts separately because the production function in URA areas may be slightly different because of the subsidy assumption that is a policy variable in MetroScope.

Residential Demand and Supply Modules – Equilibrium Iteration

We use an iterative procedure to change location prices by house type and Rzone to minimize the squared difference between total housing supply by type and total housing demand by type. To repeat equation 1 at the start we have the following procedure.

R 41.)

Select:

$$Loc Price_{Rzone,Year}^{HouseType} = [PriceIndex_{Rzone,Year2000}^{HouseType}] \times [Loc Price_{Rzone,Year}^{HouseType} Adj.factor]$$

Such that:

$$[DUActualSupply_{Rzone}^{HouseType} - HH_{Rzone}^{HouseType}]^2 = MIN$$

S.T.

$$DUActualSupply_{Rzone}^{HouseType} \geq HH_{Rzone}^{HouseType}$$

In equation R 41 we are selecting a location price adjustment factor that acts to minimize the squared difference between supply and demand by housing type and Rzone. Given a sufficiently detailed and diverse land supply in each Rzone it is possible to arrive at a close match between supply and demand over time. In practice we find it convenient to just run the model for a fixed number of iterations. This produces an adequate match between supply and demand while greatly reducing run times and making the results easily replicable.

NonResidential Model – Demand by Ezone, Employment Class and Real Estate Type

First we estimate demand by Ezone, the 15 employment types and the 6 real estate types. The demand equations are structured to be similar to “shift and share” type equations but with variables measuring relative prices and accessibilities thrown in. The train of logic here is to start with a region-wide employment estimate by type; determine the share per the base year calibration that would be located in one of the 6 real estate types and then convert that employment number to square footage using the base year calibration for square feet per employee by employment type and real estate type. The access indices act to determine the base calibration year share in a particular Ezone while the various direct price elasticity’s and cross-price elasticity’s vary amounts by Ezone, employment type and real estate type.

E 1.)

$$\begin{aligned}
 RawSqFt_{Ezone,EmpType}^{RealEstateType} &= [EmploymentTotal_{Region,EmpType}] \times [Base\%Share_{EmpType}^{RealEstateType}] \times \\
 & [BaseSqFtperEmp_{EmpType}^{RealEstateType}] \times [Location\ Price_{Ezone,t}^{RealEstateType}]^{-\beta} \times \\
 & \prod_{i=1}^6 [Location\ Price_{Ezone}^{RealEstateType}]^{\pm\alpha} \times [Location\ Price_{Ezone}^{RealEstateType}]^{-\mu} \times \\
 & \left[AccessIDX_{Ezone,EmpType}^{HH} \times \partial_{EmpType}^{HH} + AccessIDX_{Ezone,EmpType}^{AllEmp} \times \partial_{EmpType}^{AllEmp} + \right. \\
 & \left. AccessIDX_{Ezone,EmpType}^{SameEmpType} \times \partial_{EmpType}^{SameEmpType} \right]
 \end{aligned}$$

Equation E 1 allocates regional growth by employment type to each of 6 real estate types by square feet per employee and then varies those quantities as a function of the location price for that real estate type in that Ezone and the competing location prices of all other real estate types in that Ezone. Three access indices act to allocate the percentage share of each employment type that is allocated to the Ezone in tandem with the various real estate location prices. The parameters “alpha” apply to the matrix of cross elasticity’s. Here we make the standard assumption of symmetry with all cross elasticity’s summing to zero. The parameters “beta” control the change in square foot per employee from the base square foot per employee observed for the region in the calibration period. The “mu” parameter controls the location probability for the employment type. The access index parameters “capital alpha” weight the contribution of the three access indices and are set to sum to one.

E 2.)

$$RawEmp_{Ezone,EmpType}^{RealEstateType} = RawSqFt_{Ezone,EmpType}^{RealEstateType} / [BaseSqFtperEmp \times Location Price^{-\beta}]$$

E 3.)

$$AdjustFactorEmp_{Ezone,EmpType} = \frac{EmploymentTotal_{Region,EmpType}}{\sum_{Ezone} \sum_{RealEstateType} RawEmp_{EmpType}}$$

E 4.)

$$AdjustSqFt_{Ezone,EmpType}^{RealEstateType} = AdjustFactorEmp_{Ezone,EmpType} \times RawSqFt_{Ezone,EmpType}^{RealEstateType}$$

E 5.)

$$AdEmpSqFt_{Ezone,EmpType}^{RealEstateType} = AdjustFactorEmp_{Ezone,EmpType} \times$$

$$RawEmp_{Ezone,EmpType}^{RealEstateType}$$

Equations E 2 through E 5 calculate a preliminary employment estimate from the raw square footage. The preliminary employment estimate is then compared to the regional control total to determine an adjustment factor this is applied to both the square footage and employment estimates by Ezone and real estate type for each employment type.

The final component of the demand equations are the calculation of the 3 access indices noted in equation E 1 above. These access indices are calculated for each Ezone and employment type.

E 6a.)

$$RawAccessIDX_{Ezone,EmpType}^{HH} = \sum_{Ezone=j}^{72} \left[\frac{HH_{Ezone}}{[TravelTime_{Ezone} \times b_1 + TravelTime_{Ezone}^2 \times b_2]} \right]_{Ezone}$$

E 6b.)

$$AccessIDX_{Ezone,EmpType}^{HH} = RawAccessIDX_{Ezone,EmpType}^{HH} /$$

$$\sum_{Ezone=i}^{72} RawAccessIDX_{Ezone,EmpType}^{HH}$$

E 7a.)

$$RawAccessIDX_{Ezone,EmpType}^{TotalEmp} = \sum_{Ezone=j}^{72} \left[\frac{TotalEmp_{Ezone}}{[TravelTime_{Ezone} \times b_1 + TravelTime_{Ezone}^2 \times b_2]} \right]_{Ezone}$$

E 7b.)

$$AccessIDX_{Ezone,EmpType}^{TotalEmp} = RawAccessIDX_{Ezone,EmpType}^{TotalEmp} /$$

$$\sum_{Ezone=i}^{72} RawAccessIDX_{Ezone,EmpType}^{TotalEmp}$$

E 8a.)

$$RawAccessIDX_{Ezone,EmpType}^{SameEmpType} = \sum_{Ezone=j}^{72} \left[\frac{SameEmpType_{Ezone}}{[TravelTime_{Ezone} \times b_1 + TravelTime_{Ezone}^2 \times b_2]} \right]_{Ezone}$$

E 8b.)

$$AccessIDX_{Ezone,EmpType}^{SameEmpType} = RawAccessIDX_{Ezone,EmpType}^{SameEmpType} /$$

$$\sum_{Ezone=i}^{72} RawAccessIDX_{Ezone,EmpType}^{SameEmpType}$$

Equations 6 – 8 are the same equation repeated 3 different times with different weighting values for access. The first equation of the 3 access indices noted in equation E 6 above measures the access from each Ezone to the households (summed by Ezone) of the region. Access impedance here is measured by travel time and travel time squared; each adjusted by “b” parameters that are calibrated for each employment type. The “j” Ezone counter denotes summation along one row from one Ezone over all Ezones. The “i” counter denotes summation from one Ezone along one column (the sum over j column) over all Ezones. What we are doing here is summing up access for a particular Ezone and then dividing through by the total regional access for all Ezones. This operation gives the percentage share a particular Ezone has of HH access, TotalEmp access and sameEmp access. These values are then plugged into the demand equation shown as equation 1.

Nonresidential Model – Supply Side by Land Source and Real Estate Type

As noted earlier the nonresidential supply side module is slightly different than the approach used in the residential supply side module. In the residential module each of the 4 real estate types uses a land source unique to it. The nonresidential module is a bit more market oriented in that it uses but 3 land sources from which the 6 real estate types compete. Specifically, the land source of industrial supports the real estate types of manufacturing and warehousing. The land source of commercial supports the real estate types of retail/services and general office. The land source of institutional supports medical/social and government. In terms of model zoning restrictions this means that you can only build manufacturing or warehousing configured real estate on industrial land. Similarly, commercial only allows retail/service types facilities or general office. Likewise, institutional provides for medical/social or governmental type real estate.

While the real estate type is restricted to land source, there are no restrictions on the employment type that may occupy the space. Employment types are allocated to real estate types according to an observed region-wide sample average that is then varied in calibration and model operation by relative price ratios for each real estate type as denoted in the demand equation cross elasticity terms.

On the supply side we start with the land supply accounting equations.

E 9.)

$$\begin{aligned}
 TotalAcres_{Ezone, FARclass}^{Landclass} &= VacantAcres_{Ezone, FARclass, t}^{Landclass} + Re\ fillAcres_{Ezone, FARclass, t}^{Landclass} + \\
 URAAcres_{Ezone, FARclass, t}^{Landclass} &+ UGBAddAcres_{Ezone, FARclass, t}^{Landclass} + URAAddAcres_{Ezone, FARclass, t}^{Landclass}
 \end{aligned}$$

Equation E 9 states the total land supply by Ezone, land class, etc. equals the vacant, refill and urban renewal acres left over from the previous time period plus any urban renewal or UGB land added during the current forecast period.

E 10.)

$$TotalAvailableAcres_{Ezone, FARclass}^{Landclass} = TotalAcres_{Ezone, FARclass, t}^{Landclass} \times MarketFactor_{Ezone, t}^{Landclass}$$

Like the residential module, the total acres available are adjusted by a market factor that accounts for infrastructure availability, absorption rates and producer assessment of time to development.

E 11.)

$$MarketFactor_{Ezone, t}^{Landclass} = constant_{Ezone}^{Landclass}$$

$$Or := [Supply Price_{Ezone}^{RealEstateType=i \geq j}]^\alpha \times constant_{Ezone}^{Landclass}$$

$$ST := 0; \leq 1$$

Though the notation is complicated, E 11 says that the market factor is either an exogenous constant or a parameter as a function of price change from the calibration year subject to being between 0 and 1. Supply price not yet defined on the nonresidential side is covered in Equation E 15. The parameter “alpha” is typically set at 1.5. Note here too that we see the first appearance of the convention of using the higher price of the two real estate types competing in the land class to determine market factors and land prices.

E 12.)

$$BaseConCostperSqFt_{Ezone, FARclass}^{Landclass} = BaseLandCostperSqFt_{Ezone, FARclass}^{Landclass} + BaseCapitalCostperSqFt_{Ezone, FARclass}^{Landclass}$$

Equation E 12 is a straightforward accounting identity. However, note that construction costs are specified by the 3 land classes (supply side) not the 6 real estate types (demand side). This means that construction costs by FAR class and Ezone are the same for industrial/warehouse pairs, retail-service/general office pairs and medical/government pairs.

This is a simplifying assumption to avoid even more complex algorithms to allocate building between real estate types within the same land class.

E 13.)

$$PotentialSqFtCap_{Ezone, FARclass}^{Landclass} = FAR_{Ezone, FARclass}^{Landclass} \times TotalAvailableAcres_{Ezone, FARclass}^{Landclass} \times 43,560 \times NetAcres / GrossAcres$$

In Equation E 13 the FAR variable nominally ranges from .2 to 7.5 depending on FAR category. Later the variable is altered to account for changes in land class supply price. Also the reduction in gross acres to account for access, landscaping, etc. varies inversely by FAR with low FAR classes having a small reduction and high FAR classes a much larger reduction.

E 14.)

$$BasePriceSqFt_{Ezone, FARclass}^{RealEstateType} = \left[BaseAnnualRent_{Ezone, Farclass}^{RealEstateType} \right] \div \left[r \times \frac{(1+r)^n}{(1+r)^n - 1} \right]$$

Unlike residential, we seldom measure sales value for a suitable range and number of nonresidential properties. More often than not, we have annual and monthly lease rates. Equation E 14 converts the annual lease rates per square foot to a total price per square foot for a given real estate type and FAR class.

E 15.)

$$\text{Supply Price}_{Ezone}^{RealEstateType} = \frac{\text{Location Price}_{Ezone,t}^{RealEstateType}}{\text{BaseLocation Price}_{Ezone,t=0}^{RealEstateType}}$$

Equation 15 is the same form and function as the supply price equation in the residential module. It measures the location price change since the calibration year.

E 16.)

$$\text{Current PriceSqFt}_{Ezone,FARclass}^{RealEstateType} = \text{Base PriceSqFt}_{Ezone,FARclass}^{RealEstateType} \times \text{Supply Price}_{Ezone}^{RealEstateType}$$

Equation 16 updates the base price per sq. ft. from the observed calibration price to the current price. The equation substitutes for the hedonic equation used in the residential module.

E 17.)

$$\text{CurrentLand PriceRatio}_{Ezone,FARclass}^{Landclass} = \begin{cases} \text{if : } \text{Supply Price}_{Ezone}^{RealEstateType=i} \geq \text{Supply Price}_{Ezone}^{RealEstateType=j} \\ \text{then : } \left[a_o + b_1 \ln \text{Supply Price}_{Ezone}^{RealEstateType=i} \right] \\ \text{else : } \left[a_o + b_1 \ln \text{Supply Price}_{Ezone}^{RealEstateType=j} \right] \end{cases}$$

Equation 17 does two things. It converts the 2 supply prices by real estate type into one supply price by land class by taking the higher of the 2 supply prices. It then converts the supply price to a current land price ratio by using a linear function where the “b” parameter is roughly equal to the elasticity of capital and land substitution.

E 18.)

$$\begin{aligned}
\text{CurrentFAR}_{Ezone, FARclass}^{Landclass} &= \text{BaseFAR}_{Ezone, FARclass}^{LandClass} \times \left[\frac{\text{BaseCapCostSqFt}_{Ezone, FARclass}^{Landclass}}{\text{BaseLandCostSqFt}_{Ezone, FARclass}^{Landclass}} \right]^{-\delta} \\
&\times \left[\frac{\text{BaseCapCostSqFt}}{\text{BaseLandCost} \times \text{CurrentLand PriceRatio}} \right]^{-\delta}
\end{aligned}$$

Equation 18 operates across all 8 FAR class categories and updates their boundaries within lower and upper bounds to reflect the effect of price changes on the quantity of land consumed to produce a fixed amount of building capital.

E 19.)

$$\begin{aligned}
\text{CurrentCapacity}_{Ezone, FARclass}^{Landclass} &= \text{CurrentFAR}_{Ezone, FARclass}^{Landclass} \times \\
\text{TotalAvailableAcres}_{Ezone, FARclass}^{LandClass} &\times 43,560 \times \text{NetAcres} / \text{GrossAcres}
\end{aligned}$$

Equation 19 restates capacity in square feet for each FAR class after adjustment for price changes.

E 20.)

$$\begin{aligned}
\text{CurrentConCostperSqFt}_{Ezone, FARclass}^{Landclass} &= [\text{BaseLandCostperSqFt}_{Ezone, FARclass}^{Landclass} \times \\
\text{CurrentLand PriceRatio}] &+ \text{BaseCapitalCostperSqFt}_{Ezone, FARclass}^{Landclass}
\end{aligned}$$

Likewise, equation 20 adjusts the current cost per square foot for construction to reflect the revised land cost per square foot.

E 21.)

$$if : CurrentPriceSqFt_{Ezone, FARclass}^{RealEstateType} \geq CurrentConCostSqFt_{Ezone, FARclass}^{LandClass}$$

$$AdjustSq.Ft_{Ezone, FARclass}^{RealEstateType} = \sum_{i=1}^{15} AdjustSqFt_{Ezone, Emplclass}^{RealEstateType}$$

Else := 0

Equation 21 tests for each real estate type whether the current demand price is greater than or equal to the current construction cost (note this is by land class) for each of the 8 FAR classes. If it is, the algorithm sets the value at the sum of the real estate type demand for all 15 employment classes. It does this for each FAR class for which production is feasible. For those FAR classes where construction costs exceed demand price the value is set to 0.

E 22.)

$$CurrentRawSupply_{Ezone, FARclass}^{RealEstateType=i} = CurrentCapacity_{Ezone, FARclass}^{Landclass} \times \left[AdjustSqFt_{Ezone, FARclass}^{RealEstateType=i} / [AdjustSqFt_{Ezone, FARclass}^{RealEstateType=i} + AdjustSqFt_{Ezone, FARclass}^{RealEstateType=j}] \right]$$

Equation E 22 allocates the current capacity by land class between the two real estate types that compete for the capacity of that land class. This is done as a simple proportion of the demand of the two real estate types for each FAR zoning category.

E 23.)

$$CurrentAdjustSupply_{Ezone, FARclass}^{RealEstateType=i} = CurrentRawSupply_{Ezone, FARclass}^{RealEstateType=i}$$

$$if : \sum_{FarClass}^8 CurrentRawSupply_{Ezone, FarClass}^{RealEstateType=i} + \sum_{FarClass}^8 CurrentRawSupply_{EzoneFarClass}^{RealEstateType=j}$$

$$\leq \sum_{FarClass}^8 CurrentCapacity_{Ezone, FARclass}^{Landclass}$$

$$else : CurrentAdjustSupply_{Ezone, FARclass}^{RealEstateType=i} = CurrentRawSupply_{Ezone, FARclass}^{RealEstateType=i}$$

$$\sum_{Farclass}^8 CurrentCapacity_{Ezone, Farclass}^{Landclass} / \left[\sum_{FarClass}^8 CurrentRawSupply_{Ezone, FarClass}^{RealEstateType=i} + \sum_{FarClass}^8 CurrentRawSupply_{Ezone, FarClass}^{RealEstateType=j} \right]$$

Equation E 23 serves as a check on the supply allocation by real estate type to insure that the allocated capacity does not exceed the capacity of the zone for all FAR classes. If capacity is exceeded it is scaled back proportional to the allocation of raw supply to the 2 competing real estate types.

Nonresidential Demand and Supply Modules – Equilibrium Iteration

E 24.)

Select:

$$Location Price_{Ezone, Year}^{RealEstateType} = [BaseLocation Price_{Ezone, t=0}^{RealEstateType}] \times [Loc Price_{Ezone, Year}^{RealEstateType} Adj. factor]$$

Such that:

$$[CurrentActualSupply_{Ezone, t=t-1}^{RealEstateType} + CurrentAdjSupply_{Ezone, t}^{RealEstateType} - AdjSqFt_{Ezone}^{RealEstateType}]^2 = MIN$$

S.T.

$$[\text{CurrentActualSupply}_{Ezone,t=t-1}^{\text{RealEstateType}} + \text{CurrentAdjSupply}_{Ezone,t}^{\text{RealEstateType}}] \geq \text{AdjSqFt}_{Ezone}^{\text{RealEstateType}}$$

In equation E 24 we use an iterative procedure to change location prices by real estate type and Ezone to minimize the squared difference between total real estate supply by type and total real estate demand by type. This procedure is identical to the residential module.

THE END